

On the Incumbency Advantage*

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1 Introduction

In a seminal paper, [Erikson \(1971\)](#) pointed out that incumbents win reelection at strikingly high rates. This observation raised a normative concern: It raised fear that the reelection rates may be indicative of the fact that incumbents use the perquisites of office—e.g., greater access to campaign resources, gerrymandering, challenger scare off, etc—to insulate themselves from electoral threat. (See, e.g., [Fiorina \(1989\)](#).) If that is the case, then electoral accountability may be ineffective at generating good governance outcomes. [Cox and Katz \(2002, p. 7\)](#) nicely summarize the conventional wisdom:

Whenever the resources of public office are used to insulate individual politicians from electoral risk, their accountability to their constituents is weakened. . . . Thus, insulation from electoral risk of the kind suspected would, at a single stroke, debilitate the two fundamental accountability relationships of a democratic system of government.

That said, high reelection rates need not reflect the use of perquisites of office to insulate against electoral threat. As the literature has noted, these rates may instead be an artifact of the process of electoral selection. During the course of the electoral campaign, voters may learn important information about the characteristics of the candidates—e.g., quality, ideological fidelity, etc. Thus, the process of campaigning may create a situation of electoral selection, in which, on average, incumbents have characteristics that appeal to voters. As a consequence, incumbents may be more likely to be reelected. Importantly, to the extent that electoral selection is a major explanation of high incumbent reelection rates, the normative implications are reversed: Electoral environments that are characterized by high reelection rates may in fact be normatively desirable, rather than regrettable.

Figure [1.1](#) summarizes the literature’s understanding of incumbent retention rates. It points to the two mechanisms discussed. The first mechanism arises from the act of campaigning. In an open

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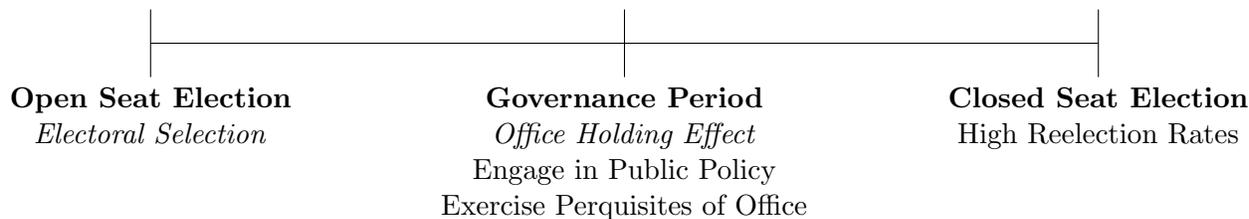


Figure 1.1: Illustration of the Literature

seat election, voters obtain information about candidates and select “good types.” As a consequence, in the subsequent closed seat election, on average, incumbents are preferred to challengers. This *electoral selection* benefits voters. The second mechanism arises during the course of holding office. While governing, the incumbent works on public policy and exercises the perquisites of office. These perquisites directly advantage her in the closed seat election, by insulating her from electoral threat. The literature views this *office holding effect* as detrimental for voters—it reduces the incumbent’s incentives to work on public policy in a way that benefits voters.

In light of these distinct normative implications, an important empirical literature has sought to isolate the effect of office holding on incumbent reelection rates. That is, the literature seeks to provide an estimate of the office holding effect on incumbent reelection rates—that is, an estimate that is purged of electoral selection. (See, e.g., [NOTE: Cites].) The idea is that such an estimate of the office holding effect serves as an estimate of how the perquisites of office impact incumbent reelection rates. Thus, the larger the estimate, the more normatively concerned we should be.

We argue that even a research design that perfectly purged the effects of electoral selection would not suffice to entail such normative conclusions. That is, isolating the effect of office holding on reelection rates does not suffice to draw normative conclusions.¹ The key observation is that there is yet an additional mechanism—one that arises during the course of governance—which serves to increase reelection rates and which can be normatively desirable.

To illustrate the alternate mechanism, note a fundamental aspect of office holding: the incumbent governs (i.e., engages in public policy). In the course of doing so, she generates governance outcomes that affect the voters’ assessment of him. So, the act of governing provides the with additional information about the incumbent—information that is not available about the challenger. This additional information is sufficient to generate incumbent electoral rates that differ systematically from election rates in open seat elections. Notice, this is an alternate candidate selection mechanism—one that enters in the course of office holding (and not over the course of campaigning).

To better understand the idea, it is important to distinguish it from others that might, at first blush, appear related. First, we focus on rational voters who use all information available to

¹Eggers (2015) points to difficulties in isolating the effect of office holding on reelection rates, given the current state of the art. (See the discussion in Section TK.) By contrast, we suppose that the empiricist can isolate the effect of office holding and reelection rates and argue that, nonetheless, we cannot draw the conventional normative conclusions.

them in forming their beliefs. At times, the additional information will lead the voter to reelect the incumbent but, at other times, the additional information will lead the voter to replace the incumbent. So, in particular, the incumbent cannot govern in a manner that is guaranteed to improve her reputation. Second, we focus on voters who cannot commit to a backroom-deal with the incumbent, whereby the incumbent is promised a vote in exchange for certain actions while in office. This again mitigates the incumbent’s ability to secure reelection. Third, we focus on voters who have instrumental preferences over governance outcomes (of the usual sort). So, in particular, they do not have, say, warm-glow preferences, which give them direct utility when voting for incumbents who have behaved well in the past.

We begin with an example that starkly illustrates the alternate candidate selection mechanism. It does so by abstracting away from the incumbent’s active role in generating governance outcomes. That is, it assumes that during the course of holding office, governance outcomes simply appear without the incumbent explicitly engaging in governance activities. On the one hand, this makes our mechanism very clear. On the other hand, it obscures important subtleties that have normative consequences. With this in mind, we then turn to a more general model in which the incumbent actively generates governance outcomes (by way of endogenous actions). This allows us to provide more comprehensive results on the normative content of the incumbency advantage. In turn, the results offer guidance regarding the nature of empirical information needed to deliver clear normative conclusions.

2 An Example

Consider the following stylized setting. There is a Voter, an initial Incumbent, and a future Challenger. Each politician can be either high or low quality. That is, each Politician P has a type θ^P , which can take on one of two values: type $\bar{\theta}$ represents high quality and type $\underline{\theta} < \bar{\theta}$ represents low quality. The probability that Politician P is high quality (i.e., $\theta^P = \bar{\theta}$) is $\pi \in (0, 1)$.

There are two governance periods, $t = 1, 2$. In each governance period, the Voter (only) observes a governance outcome. That outcome depends on the quality of the politician in office. In particular, if the politician in office in governance period t is type θ_t , the governance outcome is

$$g_t = \theta_t + \epsilon_t,$$

where ϵ_t is drawn from the standard normal distribution. In between the two governance periods, there is an election, in which the Voter can retain the Incumbent or replace her with the Challenger.

Notice, in this example, the politicians are not strategic actors. The Voter is a strategic actor, whose utility is the sum of the two governance outcomes.

Before analyzing this example, let us relate it back to the Introduction. First, we have assumed that the Incumbent and Challenger are equally likely to be high quality. In so doing, we have abstracted away from electoral selection (i.e., from the the open seat election selecting an Incumbent who is more likely to be high quality). In fact, this symmetry between candidates is illustrative

of regression discontinuity (RD) design attempts to isolate the office holding effect: They seek an idealized experiment in which the initial Incumbent is determined by a coin flip (and there is no Challenger scare-off). See, e.g., [NOTE: CITES].

Second, we have assumed that the governance outcome cannot be affected by the behavior of the Incumbent. This provides a clear exposition of our informational story. In Section TK, we allow the Incumbent to take actions that influence the governance outcomes. This is important for two reasons. First, Incumbent actions are themselves directly welfare relevant and so important for a complete normative analysis. Second, Incumbent actions endogenously affect the amount of information contained in governance outcomes and, in turn, are indirectly welfare relevant.

Third, notice that we have not explicitly modeled perquisites of office. Some perquisites of office serve to provide the Voter with information and, in that case, they work through the mechanism we are pointing to. Other perquisites of office directly impact the Voter's utility for the Incumbent relative to the Challenger. Since we have assumed that the Voter only cares about governance outcomes, we explicitly abstract away from these latter perquisites of office. To understand this subtle distinction, think about the affect of press releases. On the one hand, access to the press increases Voter information about Incumbents. This effect is captured by our analysis.² On the other hand, Incumbents can shape the tone of press releases to present a systematically positive spin; this, in turn, may directly impact the Voters' utility for the Incumbent for non-instrumental reasons. We have abstracted away from this latter effect.

In light of the above, there is only one difference between the Incumbent and the Challenger: the Voter has the opportunity to learn about the Incumbent's quality from the governance outcome. Notice that, if the Voter could also learn about the Challenger, then each candidate would win with probability one-half. That is, if the Voter had access to an independent but symmetric signal about the Challenger, then, *ex ante*, the Voter would be equally likely to obtain good news about the Incumbent versus the Challenger. So, *ex ante*, the two politicians are equally likely to be elected in the closed-seat election.

With this in mind, the **incumbency effect** is measured as the *ex ante* probability that the Incumbent wins minus one-half. If the number is strictly positive, we say that there is an incumbency advantage and, if the number is strictly negative, we say that there is an incumbency disadvantage. Thus, to understand the incumbency advantage (or disadvantage), we must understand why the *ex ante* probability that the Incumbent wins may differ from one-half. We now explain why this may be the case.

At the point where the Voter makes his electoral decision, he has observed a governance outcome g_1 . His expected second-period payoffs from Politician P is simply his conditional expectation of the Politician P 's type. So, he elects the Incumbent (resp. Challenger) whenever g_1 increases (resp. decreases) his belief that the Incumbent is high quality.

With this in mind, the Voter's electoral decision can be reduced to a problem of hypothesis testing, specifically testing $H_0 = \{\theta^I = \bar{\theta}\}$ versus $H_1 = \{\theta^I = \underline{\theta}\}$. The observed governance

²It impacts that variance of the shock ϵ_1 .

outcome g_1 can provide evidence in favor or against these hypotheses. Evidence in favor of H_0 over H_1 is good news about the Incumbent's type and so we refer to it as a **good news outcome**. Evidence in favor of H_1 over H_0 is referred to as a **bad news outcome**. If the Voter observes a good news outcome, he reelects the Incumbent; if he observes a bad news outcome, he replaces the Incumbent.

Figure TK elucidates which outcomes are good news versus bad news: The first-period governance outcome is drawn from one of two densities: $\bar{\phi}(\tilde{g}_1) \equiv \phi(\tilde{g}_1 - \bar{\theta})$ versus $\underline{\phi}(\tilde{g}_1) \equiv \phi(\tilde{g}_1 - \underline{\theta})$. These densities depend on the type of the Incumbent. The Voter is trying to figure out which density the observed outcome g_1 was drawn from. Higher governance outcomes make it more likely it came from the density associated with the high type ($\bar{\phi}$). Hence, the governance outcome is good news if and only if it is sufficiently high. In particular, as the figure makes clear, an outcome is good news if it is greater than \hat{g} where $\bar{\phi}(\hat{g}) = \underline{\phi}(\hat{g})$. Using properties of the standard normal distribution, $\hat{g} = (\bar{\theta} + \underline{\theta})/2$.

The Voter makes a Type I error if he rejects the true H_0 , i.e., he replaces an Incumbent who is in fact high quality. The probability of a Type I error is

$$\alpha \equiv \Pr(\text{Type I error}) = \Pr(g < \hat{g} \mid \theta^I = \bar{\theta}).$$

The Voter makes a Type II error if he fails to reject the false H_0 , i.e., he reelects the Incumbent who is in fact low quality. The probability of a Type II error is

$$\beta \equiv \Pr(\text{Type II error}) = \Pr(g \geq \hat{g} \mid \theta^I = \underline{\theta}).$$

The shaded regions in Figure TK correspond to the probabilities of Type I and Type II errors. It illustrates two important properties. First, because the distribution of ϵ is symmetric and \hat{g} is the midpoint of $\bar{\theta}$ and $\underline{\theta}$, Type I and Type II errors are equally likely. Second, these probabilities are each strictly positive and strictly less than one-half. So, $\alpha = \beta \in (0, 1/2)$.

The Incumbent is reelected if she is a high quality and there is no Type I error or if she is a low quality and there is Type II error. Thus, the probability that the Incumbent is reelected is:

$$\begin{aligned} \Pr(\text{Reelect Incumbent}) &= \pi \Pr(g \geq \hat{g} \mid \theta^I = \bar{\theta}) + (1 - \pi) \Pr(g \geq \hat{g} \mid \theta^I = \underline{\theta}) \\ &= \pi(1 - \alpha) + (1 - \pi)\beta \end{aligned}$$

Using the fact that $\alpha = \beta$, the incumbency effect is

$$\begin{aligned} \mathcal{IE} &= \pi(1 - \alpha) + (1 - \pi)\alpha - \frac{1}{2} \\ &= \frac{1}{2}(2\pi - 1)(1 - 2\alpha) \end{aligned}$$

Since $\alpha \in (0, \frac{1}{2})$, the third term is strictly positive. Hence, the incumbency effect is strictly increasing in the prior probability any politician is a high type, π . Moreover, it is zero if and only

if $\pi = \frac{1}{2}$. So, our example predicts an incumbency disadvantage whenever the pool of candidates contains more low quality than high quality candidates (i.e., $1/2 > \pi$) and an incumbency advantage whenever the pool of candidates contains more high quality than low quality candidates (i.e., $\pi > 1/2$).³

In our example, the incumbency effect arises *only* from the fact that the Voter learns about the Incumbent (but not the Challenger) by observing the outcome of governance. High types are more likely to generate good news outcomes and so more likely to be reelected. The more high types there are in the pool of candidates, the more likely it is that the Incumbent will generate good news and be reelected. Hence, the larger π , the higher the reelection rate of Incumbents. If the pool is made up equally of high types and low types, then there is no systematic incumbency effect. If the pool has a majority of high (respectively, low) types, there is a systematic incumbency advantage (respectively, disadvantage).

Before concluding, it is important to note that the analysis here is quite different from that in Eggers (2015). In particular, he is concerned with a distinct question from the one studied here: Are RD estimates of the incumbency effect, in fact, purged of electoral selection? He points out that if, in the open seat election, the pool of candidates has more high types than low types—that is, if $\pi > 1/2$ —then RD estimates do not capture the idealized experiment discussed on page 4.⁴ When $\pi > 1/2$, then it is more likely that there will be two high quality versus low quality candidates in the open seat election. As a consequence, when the open seat race is close (as in the RD design), the Incumbent is more likely to be a high quality candidate. In that case, RD estimates are not purged of electoral selection. Our focus is different. We argue that, even if the empirical literature provides an estimate of the incumbency effect that is purged of electoral selection, there would still be questions about the normative interpretation. The reason is that, over the course of governance, the Voter can further learn about the Incumbent’s quality.⁵

3 The Model

We now extend the example, so that the politician in office can endogenously choose effort to influence the governance outcome. We only discuss changes to the original example; all other assumptions remain the same.

The politicians do not know their type. In each governance period, the Politician in office chooses effort in $A \subseteq \mathbb{R}_+$. The set A is closed and has a smallest element \underline{a} . The Politician in office now impacts the governance outcome in two ways—her type and the level of effort her chooses. Specifically, there is a **production function** $f : A \times \Theta \rightarrow \mathbb{R}_+$ that is strictly increasing in effort (a) and type (θ). So, if, in period t , the Politician in office chooses effort level a , is of type θ , and

³Some empirical papers do, in fact, find a negative incumbency advantage (cites TK). Most such findings are in less-established democracies. This is interesting, in light of our results, since one might, in fact, believe that, in less-established democracies, the pool of candidates is of lower quality than in well-established democracies.

⁴See the two-type example on page 9 of Eggers (2015).

⁵Given his distinct focus of his paper, Eggers (2015) does not model governance and so does not make this point.

the random shock is ϵ_t , the level of public goods produced in that period is $g_t = f(a, \theta) + \epsilon_t$. As above, each ϵ_t is drawn from the standard normal distribution.⁶

Each Politicians' payoffs depend on both a benefit from holding office and the effort chosen while in office. The benefit from holding office is given by $B > 0$. The cost of effort is given by a function $c(\cdot)$, where $c(\cdot)$ is strictly increasing in the effort level and $B > c(\underline{a}) = 0$. A Politician's payoff in governance period t is 0 if she is not in office and $B - c(a_t)$ if she is in office and chooses effort a_t . A Politician's payoffs are given by the sum of her payoffs in each governance period. The Voter's payoff remains the sum of the two governance outcomes.

An important aspect of our analysis will involve how a politician's effort and type interact to produce governance outcomes.

Definition 3.1.

(i) Effort and type are **complements** if, for any $a_{**} > a_*$, $f(a_{**}, \bar{\theta}) - f(a_{**}, \underline{\theta}) \geq f(a_*, \bar{\theta}) - f(a_*, \underline{\theta})$.

(ii) Effort and type are **substitutes** if, for any $a_{**} > a_*$, $f(a_*, \bar{\theta}) - f(a_*, \underline{\theta}) \geq f(a_{**}, \bar{\theta}) - f(a_{**}, \underline{\theta})$.

If effort and type are complements, then an increase in type has a larger impact on productivity when the Politician chooses higher over lower effort. If effort and type are substitutes, then an increase in type has a smaller impact on productivity when the Politician chooses a higher over lower effort.

4 Properties of Equilibrium

In this section, we point to basic properties of equilibrium, which will be useful in our subsequent analysis. We focus on pure-strategy Perfect Bayesian Equilibrium.

The Voter's Reelection Decision In the second governance period, there are no electoral benefits from choosing costly effort. As such, the Politician in office will choose the lowest possible level of effort, i.e., $a_2 = \underline{a}$, independent of the history. So, the Voter's electoral decision depends only on his expectation about which Politician is most likely to be a high type.

Just as in the example (Section 2), the Voter's electoral decision can be reduced to a problem of hypothesis testing, specifically testing $H_0 = \{\theta^I = \bar{\theta}\}$ versus $H_1 = \{\theta^I = \underline{\theta}\}$. Suppose the Voter believes the Incumbent took the first-period action a . Then he believes the governance outcome is drawn from one of two densities: one possibility is $\bar{\phi}(g_1|a) \equiv \phi(g_1 - f(a, \bar{\theta}))$ and the second possibility is $\underline{\phi}(g_1|a) \equiv \phi(g_1 - f(a, \underline{\theta}))$. The density $\bar{\phi}(\cdot|a)$ has a mean of $f(a, \bar{\theta})$ and the density $\underline{\phi}(\cdot|a)$ has a mean of $f(a, \underline{\theta})$. Define the mid-point of these two means as

$$\hat{g}(a) = \frac{f(a, \bar{\theta}) + f(a, \underline{\theta})}{2}.$$

⁶This assumption is not needed for our analysis. In particular, our results can be extended to the case where the density is symmetric about 0 and the likelihood ratio associated with ϕ is onto \mathbb{R}_+ , and has strictly positive derivative.

Any governance outcome greater than $\hat{g}(a)$ is evidence in favor of H_0 and against H_1 , i.e., a **good news outcome**. Any governance outcome less than $\hat{g}(a)$ is evidence in favor of H_1 and against H_0 , i.e., a **bad news outcome**. Hence, the Voter reelects the Incumbent if g_1 is greater than $\hat{g}(a)$ and replaces her if g_1 is less than $\hat{g}(a)$. (See Example 2.1 in [Ashworth, Bueno de Mesquita and Friedenberg \(forthcoming\)](#), henceforth [ABF](#).)

The Incumbent's First-Period Choice Suppose the Incumbent chooses the action a and the Voter expects the Incumbent to choose action a_* . The Incumbent is reelected (resp. replaced) if the level of public goods observed, $f(a, \theta^I) + \epsilon_1$, exceeds (resp. falls short of) the Voter's threshold $\hat{g}(a_*)$. The probability that the Incumbent is reelected if she chooses a when the Voter expects her to choose a_* , written $\Pr(a|a_*)$, is then the probability that $\epsilon_1 \geq \hat{g}(a_*) - f(a, \theta^I)$. Writing Φ for the CDF of the standard normal distribution, this probability is

$$\Pr(a|a_*) = \pi [1 - \Phi(\hat{g}(a_*) - f(a, \bar{\theta}))] + (1 - \pi) [1 - \Phi(\hat{g}(a_*) - f(a, \underline{\theta}))]. \quad (1)$$

There is a pure-strategy equilibrium in which the Incumbent chooses the first-period effort level a_* if and only if

$$B \Pr(a_*|a_*) - c(a_*) \geq B \Pr(a|a_*) - c(a)$$

for all effort levels $a \in A$.

Comparative Statics In the following sections, we will explore how the incumbency effect and voter welfare are related to the Incumbent's equilibrium effort. It is thus of interest to relate comparative statics of effort to primitives of the model. We do so here in terms of the Incumbent's benefit of reelection. This is a salient policy parameter that is often studied in the literature on political accountability ([Gagliarducci and Nannicini, 2013](#)).

A first point to make is that this model can have multiple equilibria that differ in the Incumbent's choice of effort. (See Section 5 of [ABF](#).) To handle this possible multiplicity, we state a comparative static result in terms of the the extremal equilibrium efforts. For each benefit of reelection B , write $a_H(B)$ for the largest equilibrium effort at benefit of reelection B , and $a_L(B)$ for the lowest equilibrium effort at benefit of reelection B .

Proposition 4.1. *Suppose that:*

- $A = \mathbb{R}_+$;
- f is concave and differentiable in a , with $\frac{\partial f}{\partial a} > 0$; and
- c is strictly convex and differentiable, with $c'(0) = 0$ and $\lim_{a \rightarrow \infty} c'(a) = \infty$.

Then, for each B , there are largest and smallest [candidates for] equilibrium efforts, $a_H(B)$ and $a_L(B)$. Moreover, both of these are strictly increasing in B .

Proof. If the Voter expects the Incumbent to choose effort a_* , then he will use the reelection threshold $\hat{g}(a_*)$. Given this, the Incumbent solves:

$$\max_{a \in A} B \left[\pi \left(1 - \Phi(\hat{g}(a_*) - f(a, \bar{\theta})) \right) + (1 - \pi) \left(1 - \Phi(\hat{g}(a_*) - f(a, \underline{\theta})) \right) \right] - c(a).$$

The first-order necessary condition for an interior a is:

$$B \left[\pi \phi(\hat{g}(a_*) - f(a, \bar{\theta})) \frac{\partial f}{\partial a}(a, \bar{\theta}) + (1 - \pi) \phi(\hat{g}(a_*) - f(a, \underline{\theta})) \frac{\partial f}{\partial a}(a, \underline{\theta}) \right] - c'(a) = 0.$$

If a_* is an equilibrium effort, then this first-order condition is satisfied at $a = a_*$:

$$\Delta(a_*, B) \equiv B \phi(\iota(a_*)) \left(\pi \frac{\partial f}{\partial a}(a_*, \bar{\theta}) + (1 - \pi) \frac{\partial f}{\partial a}(a_*, \underline{\theta}) \right) - c'(a_*) = 0.$$

Since $\frac{\partial f}{\partial a}(0, \theta) > 0$ and $c'(0) = 0$, $\Delta(0, B) > 0$. Since $\lim_{a \rightarrow \infty} c'(a) = \infty$, there is an \bar{a} such that $a \geq \bar{a}$ implies $\Delta(a, B) < 0$. Δ is continuous in a_* and strictly increasing in B , so Theorem 1 in [Milgrom and Roberts \(1994\)](#) implies that, for each B , there are a largest and smallest solutions, $a_H(B)$ and $a_L(B)$, to the equation. Moreover, each of these is strictly increasing in B . ■

5 The Incumbency Advantage

The incumbency effect captures the incremental likelihood of being reelected relative to the likelihood of being elected in a race of equal-footing. The key is that, now, the likelihood of being reelected depends on the first-period equilibrium level of effort.

Fix an equilibrium where the Incumbent chooses a first-period level of effort a_* . Write $\Pr(\text{reelection} | a_*)$ for the *ex ante* probability of reelection given that the first-period equilibrium action is a_* . Then, the **incumbency effect** is measured as

$$\mathcal{IE}(a_*) = \Pr(\text{reelection} | a_*) - \frac{1}{2}.$$

Notice, the incumbent's first-period effort level a_* influences the *ex ante* probability of reelection and, in so doing, influences the incumbency effect.

To understand this fact, refer back to the example (Section 2). The Incumbent is reelected if either there is no Type I error or there is a Type II error. Now, the likelihood of these errors depends on a_* . Specifically, the probability of a Type I error is

$$\alpha(a_*) \equiv \Pr(f(a_*, \bar{\theta}) + \epsilon_1 < \hat{g}(a_*)) = \Phi \left(-\frac{f(a_*, \bar{\theta}) - f(a_*, \underline{\theta})}{2} \right).$$

and the probability of a Type II error is

$$\beta(a_*) \equiv \Pr(f(a_*, \underline{\theta}) + \epsilon_1 \geq \hat{g}(a_*)) = 1 - \Phi \left(\frac{f(a_*, \bar{\theta}) - f(a_*, \underline{\theta})}{2} \right).$$

Using properties of the standard normal distribution, $\alpha(a_*) = \beta(a_*)$.

Notice, both the probability of a Type I error and the probability of a Type II error are decreasing in the variable

$$\iota(a_*) \equiv \frac{f(a_*, \bar{\theta}) - f(a_*, \theta)}{2}.$$

To understand this fact, recall, the Voter is trying to determine whether the true density is $\bar{\phi}(\cdot|a_*)$ versus $\underline{\phi}(\cdot|a_*)$. When $\iota(a_*)$ is larger, the difference between the means of these densities is larger. This makes it easier to determine which density the governance outcome came from. As such, it is less likely that the Voter will make a mistake.

In fact, $\iota(a_*)$ captures how [Blackwell \(1951\)](#) informative the signal is about the Incumbent's type. Proposition 3.1 in [ABF](#) shows the following result: The first-period level of effort a_{**} provides more information about the Incumbent's type relative to the first-period level of effort a_* if and only if $\iota(a_{**}) > \iota(a_*)$. In light of this, we refer to $\iota(\cdot)$ as *informativeness*. Notice that, when effort and type are complements, an increase in effort leads to an increase in informativeness. But, when they are substitutes, an increase in effort leads to a decrease in informativeness.

Changing the equilibrium level of effort serves to change the Voter's information about the Incumbent's type and, in so doing, impacts the probability of reelection and the incumbency effect. In particular,

$$\begin{aligned} \mathcal{IE}(a_*) &= \pi(1 - \alpha(a_*)) + (1 - \pi)\alpha(a_*) - \frac{1}{2} \\ &= \pi + (1 - 2\pi)\alpha(a_*) - \frac{1}{2} \\ &= (2\pi - 1) \left(\Phi(\iota(a_*)) - \frac{1}{2} \right) \end{aligned} \tag{2}$$

Notice, when $\pi = 1/2$, there is no incumbency effect. Moreover, the incumbency effect is increasing in π . (To see this observe that $\iota(a_*) > 0$ and so $\Phi(\iota(a_*)) > 1/2$; from this, the second term is positive.) Thus, much as in the example of [Section 2](#):

Proposition 5.1. *There is an incumbency advantage if $\pi > 1/2$ and an incumbency disadvantage if $1/2 > \pi$.*

Now consider a situation where the first-period equilibrium level of effort changes. This change in the Incumbent's behavior serves to change the informativeness of the Voter's signal which, in turn, serves to impact the incumbency effect. However, the impact on the incumbency effect depends on the pool of candidates. Refer to [Equation \(2\)](#) and note that $\pi > 1/2$ if and only if $2\pi - 1 > 0$. So, if $\pi > 1/2$, an increase in informativeness serves to increase the incumbency effect. However, if $\pi < 1/2$, then an increase in informativeness serves to decrease the incumbency effect.

Recall, higher levels of effort impact informativeness differently, if effort and type are complements versus if they are substitutes. Thus, the impact of a change in effort on the incumbency effect will depend both on the nature of production and the pool of candidates. This impact is summarized by [Table 1](#).

	Complements	Substitutes
$\pi > \frac{1}{2}$	$\mathcal{IE}(a)$ increasing in a	$\mathcal{IE}(a)$ decreasing in a
$\pi < \frac{1}{2}$	$\mathcal{IE}(a)$ decreasing in a	$\mathcal{IE}(a)$ increasing in a

Table 1: Incumbency Effect

6 Voter Welfare

Voter welfare is the sum of the expected first-period welfare and the *ex ante* expected second-period Voter welfare. Each of these are impacted by the Incumbent's first-period level of effort, a_* . In particular, the expected first-period welfare is

$$\text{VW}_1(a_*) = \pi f(a_*, \bar{\theta}) + (1 - \pi) f(a_*, \underline{\theta})$$

and the *ex ante* expected second-period welfare is

$$\text{VW}_2(a_*) = \Pr(\theta_2 = \bar{\theta} | a_*) f(a, \bar{\theta}) + (1 - \Pr(\theta_2 = \bar{\theta} | a_*)) f(a, \underline{\theta}).$$

Here, $\Pr(\theta_2 = \bar{\theta} | a_*)$ represents the *ex ante* (equilibrium) probability that the Politician in office in the second period is of type $\bar{\theta}$, given that the first-period effort level is a_* .

Notice that the first-period welfare is necessarily increasing in the Incumbent's first-period level of effort a_* . The second-period welfare is increasing in the Incumbent's first-period level of effort a_* if and only if $\Pr(\theta_2 = \bar{\theta} | a_*)$ is increasing in a_* . Thus, to understand how the first-period effort level a_* impacts the second-period welfare, we must understand how it impacts $\Pr(\theta_2 = \bar{\theta} | a_*)$.

Following the logic in the example (Section 2),

$$\begin{aligned} \Pr(\theta_2 = \bar{\theta} | a_*) &= \pi(1 - \alpha(a_*)) + \pi^2 \alpha(a_*) + (1 - \pi)\pi(1 - \beta(a_*)) \\ &= \pi(2 - \pi) - 2\pi(1 - \pi)\alpha(a_*). \end{aligned} \tag{3}$$

Notice, as the informativeness increases, $\alpha(\cdot)$ decreases. This, in turn, increases the probability of selecting a high quality candidate in the second-period and so serves to increase *ex ante* expected second-period welfare.

	Complements	Substitutes
VW_1	increasing in a	increasing in a
VW_2	increasing in a	decreasing in a

Table 2: Welfare

The impact of a change in effort on welfare, thus, depends on whether effort and type are complements versus substitutes. Refer to Table 2. When effort and type are complements, increasing effort serves to increase informativeness. With this, higher levels of first-period effort increases

both first- and second-period welfare—so total welfare necessarily increases. However, when they are substitutes, increasing effort serves to decrease informativeness. With this, higher levels of first-period effort increase first-period welfare and decrease second-period welfare. Example 3.1 in [ABF](#) shows that total welfare can decrease. However, increasing first-period effort increases total welfare, if the pool of candidates has a sufficiently larger number or summer number of high quality candidates. (See Proposition 3.1 in [ABF](#).⁷)

7 Voter Welfare and the Incumbency Effect

Comovement of Voter Welfare and Incumbency Advantage with Effort Variation Recall that expected first-period voter welfare, $\text{VW}_1(a)$, is increasing in a . Whether expected second-period voter welfare is increasing or decreasing in a depends on the technology.

Proposition 7.1. *Fix $a_{**} > a_*$. If effort and type are complements, then $\text{VW}_2(a_{**}) \geq \text{VW}_2(a_*)$.*

Proof. Complements implies $\iota(a_{**}) > \iota(a_*)$. Then use Lemma ??.

Proposition 7.2. *Fix $a_{**} > a_*$, and suppose effort and type are substitutes. There exist $\underline{\pi}[a_*, a_{**}], \bar{\pi}[a_*, a_{**}] \in (0, 1)$ so that the following are equivalent:*

- (i) $\pi \in (0, \underline{\pi}[a_*, a_{**}]) \cup (\bar{\pi}[a_*, a_{**}], 1)$.
- (ii) $\text{VW}_1(a_{**}) + \text{VW}_2(a_{**}) \geq \text{VW}_1(a_*) + \text{VW}_2(a_*)$.

Proof. Proposition 3.1 in AIE.

Proposition 7.3. *Fix $a_{**} > a_*$.*

- (i) *If effort and type are complements and $\pi \geq \frac{1}{2}$, then $\text{IA}(a_{**}) \geq \text{IA}(a_*)$.*
- (ii) *If effort and type are complements and $\pi \leq \frac{1}{2}$, then $\text{IA}(a_{**}) \leq \text{IA}(a_*)$.*
- (iii) *If effort and type are substitutes and $\pi \geq \frac{1}{2}$, then $\text{IA}(a_{**}) \leq \text{IA}(a_*)$.*
- (iv) *If effort and type are substitutes and $\pi \leq \frac{1}{2}$, then $\text{IA}(a_{**}) \geq \text{IA}(a_*)$.*

Proof. Consider the case of complements and $\pi \geq \frac{1}{2}$; the other cases are similar.

Complements implies that $\iota(a_{**}) \geq \iota(a_*)$. $\pi \geq \frac{1}{2}$ if and only if $(2\pi - 1) \geq 0$. Thus:

$$\text{IA}(a_{**}) = (2\pi - 1) \left(\Phi(\iota(a_{**})) - \frac{1}{2} \right) \geq (2\pi - 1) \left(\Phi(\iota(a_*)) - \frac{1}{2} \right) = \text{IA}(a_*).$$

■

⁷More precisely: For any first-period effort levels a_* and $a_{**} > a_*$, there are $\underline{p}[a_*, a_{**}], \bar{p}[a_*, a_{**}] \in (0, 1)$ so that, if $\pi \in (0, \underline{p}[a_*, a_{**}]) \cup (\bar{p}[a_*, a_{**}], 1)$, total welfare increases when the effort level increases from a_* to a_{**} . (Note, $\underline{p}[a_*, a_{**}]$ may—but need not—be less than $\bar{p}[a_*, a_{**}]$.) When A is finite, this implies that there exists $\underline{p}, \bar{p} \in (0, 1)$ so that, if $\pi \in (0, \underline{p}) \cup (\bar{p}, 1)$, increasing the effort level increases total welfare.

Comparing Propositions 7.1, 7.2, and 7.3 shows that the incumbency advantage is not, in general, a valid measure of voter welfare.

Consider first the case of complements. A shift from an equilibrium with low effort to one with higher effort is associated with an increase in both components of Voter welfare. If $\pi > \frac{1}{2}$, the shift is also associated with an increase in the incumbency advantage. This is the opposite of the relationship between Voter welfare and incumbency advantage suggested by the literature's discussion of "the advantages of incumbency".

Now consider the case of substitutes. A shift from an equilibrium with a low effort to one with a higher effort is associated with an increase in the sum of components of Voter welfare if π is close enough to 1. But in this case, Proposition 7.3 says that the change is associated with a smaller incumbency advantage.

Comparing the previous two paragraphs shows that a shift from an equilibrium with low effort to one with higher effort can be associated with changes in Voter welfare and the incumbency advantage that go in the same direction, or in opposite directions. (Note that in all cases considered, the incumbency advantage is positive. Thus there is no obvious empirical criterion to tell which case we are in.)

References

- Ashworth, Scott, Ethan Bueno de Mesquita and Amanda Friedenberg. forthcoming. “Accountability and Information in Elections.” *American Economic Journal* .
- Blackwell, David. 1951. The Comparison of Experiments. In *Proceedings, Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: University of California Press pp. 93–102.
- Cox, Gary W. and Jonathan N. Katz. 2002. *Elbridge Gerry’s Salamander: The Electoral Consequences of the Reapportionment Revolution*. New York: Cambridge University Press.
- Eggers, Andy. 2015. “Quality-Based Explanations of Incumbency Effects.” Forthcoming in the *Journal of Politics*.
- Erikson, Robert S. 1971. “The Advantage of Incumbency in Congressional Elections.” *Polity* 3(3):395–405.
- Fiorina, Morris P. 1989. *Congress: Keystone of the Washington establishment*. Yale University Press.
- Gagliarducci, Stefano and Tommaso Nannicini. 2013. “Do Better Paid Politicians Perform Better? Disentangling Incentives from Selection.” *Journal of the European Economic Association* 11(2):369–398.
- Milgrom, Paul and John Roberts. 1994. “Comparing Equilibria.” *The American Economic Review* 84(3):441–459.