Dynamic Pivotal Politics[1]

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Abstract

We build a parsimonious dynamic model of lawmaking, and show that unlike in static models, policy makers' voting behavior may not be sincere. Instead, it depends on the details of political institutions and policy makers' expectations about the future economic and political environment. For institutions with checks and balances, policy makers vote in a more polarized way than implied by their idelogies and may exhibit a policy bias. These effects increase with the degree of consensus required by the institution, the volatility of the economic environment and the expected ideological polarization of the future policy makers. We draw conclusions for the consequences of increasing a hurdle for deficit inducing policy changes, redistricting, and midterm elections.

1 Introduction

Theoretical models of lawmaking integrate legislators' preferences and institutions to understand legislative outcomes (Krehbiel 1998, Brady and Volden 2006, Cox and McCubbins 2005, Chiou and Rothenberg 2003). These models typically assume that legislators vote sincerely. Sincere voting implies that institutions affect only agenda setting and how votes are aggregated. This independence of institutions and voting behavior allows theorists to analyze the role of institutions taking legislators' voting behavior as fixed. Likewise, it allows empiricists to recover legislators' preferences from roll call votes independent of the institutional settings (e.g., Poole and Rosenthal 1985, Heckman and Snyder 1997)

These models of lawmaking, however, are inherently static. They take the current status quo as exogenous and overlook the legislative process that led to it. In reality, the legislative process is inherently dynamic, with legislators periodically revising policies to respond to changing circumstances. Thus, the current status quo is inherited from previous votes, and is therefore endogenous. As highlighted by game theoretic literature (Baron 1996, Kalandrakis 2004, Riboni and Ruge Murcia 2008, Dziuda and Loeper 2016), the endogeneity of the status quo can distort legislators's behavior in important ways. The goal of this paper is to introduce dynamics into a parsimonious model of lawmaking, and investigate how the endogeneity of the status quo affects the main conclusions of the static literature.

In this paper, we analyze an infinite-horizon extension of an otherwise parsimonious model of lawmaking. A set of policy makers repeatedly choose between two policies, and the policy implemented in a given period can be revised in the next period only if a decisive coalition under the prevailing institution agrees to do so. Every period, policy makers' preferences over these policies are subject to shocks, which represent any variables that affect desirability of various policies, such as economic indicators, demographic trends, international situation or simply the vagaries of public opinion.

We show that, consistently with the findings in Dziuda and Loeper (2016), under all institutions with some degree of checks and balances, policy makers do not vote sincerely.[1] Specifically, legislators' voting behavior is distorted in two systematic ways. First, the voting behavior exhibits *strategic polarization*: relative to sincere voting, any two legislators disagree more frequently. As a result, the legislature gridlocks more frequently. Second, policy makers may also exhibit *policy bias*; that is, the voting behavior of legislature can be tilted towards a particular policy on average.

The intuition for these distortions is as follows. For institutions with checks and balances,

[1] Dziuda and Loeper (2016) analyzed only unanimity, while we allow for a broad class of voting rules. We describe in detail how this paper relates to Dziuda and Loeper (2016) in Section 2.

one can identify two distinct policy makers, whose agreement is needed for a policy change (Krehbiel 1998, Brady and Volden 2006). Following Krehbiel, we will call these legislators *pivots*. Since the status quo stays in place whenever the pivots disagree, policy makers have incentives to affect which policy is the status quo. In the dynamic model, they can affect tomorrow's status quo by distorting their today's votes. Rightist and leftist policy makers disagree on which policy should be the status quo. As a result, relative to their sincere preferences, the former vote more frequently for the conservative policy and the latter more frequently for the liberal policy, leading to an increase in gridlock. And since legislators on average may prefer one policy to be the status quo, legislature as a whole may exhibit a policy bias in the direction of this policy.

One might argue that our theory is observationally indistinguishable from the static theory of voting, the only difference being that the static models treat legislators' voting behavior as the sincere expression of their intrinsic preferences, whereas our model interprets it as a more complex equilibrium object. However, we show that in the dynamic model, voting behavior is no longer independent of the institutions. This implies that when analyzing the impact of institutional changes, one cannot take as given the preferences induced from the current or past votes.

To see how the static and the dynamic models differ in their predictions of the impact of an institutional change, consider an increase in the size of coalitions required to implement a liberal policy change. A static model makes two predictions. First, a more rightist policy maker becomes pivotal for a liberal policy change. This means that the pivots are more ideologically polarized, which increases gridlock. Second, a conservative policy change becomes more frequent relative to the liberal policy change. Our analysis shows that these predictions need to be qualified. First, the static model underestimates the inertial effect of such institutional change. To see this, note that an increase in the likelihood of future gridlock, increases incentives to affect the future status quo. As a result, legislators vote in a more polarized way, increasing the gridlock further. Second, due to the strategic policy bias, legislature as a whole may vote in a more liberal way than before the institutional change. We show that this strategic effect can be so strong that the incidence of liberal policies in the long run may be higher than before. This follows because liberal legislators become less willing to move policies in the conservative direction when needed, by fear of not being able to revert to more liberal policies once the environment changes.

Our model reveals another determinant of legislators' behavior that by assumption is absent from static models, namely legislators' expectations about the future economic and political environment. We show that a legislature may vote quite differently when considering issues which differ in the degree of uncertainty about the future benefits of the policies, and

two seemingly identical legislatures may vote differently when expecting different future political environment. Specifically, we first show that policy makers vote in a more polarized manner on issues that are subject to frequent shocks. That is, there is more gridlock and hence policy inertia precisely in domains in which the policy should respond quickly to a rapidly changing environment. A prominent example of a policy domain that is subject to frequent shocks is taxation: tax policies must react to varying factors such as the business cycle, changes in the countries' fiscal sustainability, or voters' mood swings. Our findings suggest that procedural rules that limit the power of the filibuster, such as the reconciliation process, can be highly desirable in such domains.

Second, we show that policy makers' behavior depends on their expectations about future electoral outcomes. For example, if the future governments are likely to be united, in which case both future pivots have identical preferences, the incentives of the current policy makers to defend the favorable status quo are low, and so they vote more in line with their sincere preferences. On the other hand, if they expect future governments to be divided, they bias their voting behavior significantly. This leads to high polarization when the current government is divided, but also to distorted voting when the current government is united. For example, a united conservative government may decrease taxes even in times of high budget deficit in order to put themselves in a better negotiating position in case the future government is divided.

One of the puzzles of recent years has been an increase in polarization of legislators in the U.S. Congress as inferred from their voting behavior (see, e.g., McCarty et al. 2008). The goal of this paper is to analyze lawmaking in the simplest dynamic setting; hence, we abstract from many aspects of lawmaking that may speak to this increase in polarization. Nevertheless, our results suggest three possible factors that may have contributed to it.[2] First, concurrently with the raise in measured polarization, the filibuster—once an infrequently used tool reserved for the most important and controversial bills—has become a routine practice in American politics (see Table 3, and also Koger 2010, and chapter 5 of Binder and Smith 1997). Some political scientists and pundits blame the more frequent use of the filibuster on the increase in legislators' ideological polarization (Krehbiel 1998, McCarty 2007). Our results suggest a possible reverse causality: the change in institutional practice introduced more checks and balances and hence made more extreme legislators' pivotal. This, according to our results, should result in an increase of strategic polarization among legislators. Second, since the 1970s, divided governments have become increasingly

[2] Political scientists have proposed several possible explanations for this phenomenon, such as electorate polarization, party primaries, or economic inequality (see, e.g., Barber and McCarty 2010). Our results solely suggest that independently of the evolution of legislators' ideologies, the change in institutional practice since the 70s may have triggered an increase in strategic polarization.

common in the U.S. Arguably, this should have affected policy makers' expectations about future gridlock. According to our results, such a change in expectations should have pushed the legislators to vote in a more polarized way, even during the times of united government. And finally, even if ideological polarization is indeed on the raise, our results suggest that the studies might have overestimated the magnitude of such increase. In the dynamic model, any increase in ideological polarization is magnified by an increase in strategic polarization.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the basic model. Section 4 first characterizes the equilibrium in the benchmark static game (Section 4.1) and then characterizes equilibria in the dynamic game (Section 4.2). Section 5 analyzes the impact of increasing the degree of consensus required by the voting institution the equilibrium behavior. Section 6 analyzes how the behavior of policy makers varies with the expectations they hold about the future economic and political environment, including future electoral outcomes. Section 7 concludes. All proofs are in the appendix.

2 Related literature

To be added.

3 The model

Players and policies

A set of policy makers $\mathcal{N} = \{1, ..., n, ..., N\}$ are in a relationship that lasts for infinitely many periods. In each period, they must decide which of the two policies L and R to implement, where L can be interpreted as a liberal policy such as a high tax rate, high government spending, or high level of protection of domestic industries, and R as its conservative counterpart.

Payoffs

Each policy maker $n \in \mathcal{N}$ maximizes her expected discounted sum of period payoffs with discount factor $\delta \in (0, 1)$. Her payoff gain from implementing policy R instead of L in some period $t \in \mathbb{N}$ is denoted by $\theta_n(t)$. Hence, if $\theta_n(t)$ is positive (negative), policy maker n prefers the conservative policy R (the liberal policy L) in period t. One can view this specification as a stylized dynamic extension of the canonical spatial model where $\theta_n(t)$ is the peak of policy maker n's single-peaked utility function.[3] We refer to $\theta_n(t)$ as *the sincere*

[3] To see this, note that it is equivalent to assuming that the payoff for n from implementing $x \in \{L, R\}$ in some period t is $-(x - \theta_n(t))^2$, with $R = -L = 1/4$.

preferences of policy maker n in period t, and to $\theta(t) \doteq (\theta_1(t), ..., \theta_N(t))$ as the state of nature in period t.

The stochastic process $\{\theta(t) : t \in \mathbb{N}\}$ that governs the evolution of the state is assumed to be i.i.d. across periods, and distributed according to an integrable probability distribution P that satisfies the following property.

Assumption 1 *For all $t \in \mathbb{N}$, $\theta_1(t) < ... < \theta_N(t)$ with probability one, and for any two policy makers $n, m \in \mathcal{N}$ with $n < m$, $\theta_n(t) < 0 < \theta_m(t)$ with positive probability.*

Assumption 1 states that even though policy makers' preferences over a given issue evolve over time, their ideological rank order on that issue remains constant. Specifically, those with lower indices are more leftist than those with higher indices. Therefore, whenever two policy makers disagree on a given issue, the relatively more leftist on that issue always prefers L and the relatively more rightist on that issue prefer R. This assumption is in line with Poole and Rosenthal (1991), who say "Of course, Congress as a whole may adapt by, for example, moving to protectionism when jobs are lost to foreign competition. But as such new items move onto the agenda, their cutting lines will typically be consistent with the preexisting, stable voting alignments".[4]

The following definition characterizes a simple environment in which Assumption 1 is satisfied. We will use this environment in the numerical examples of this paper.

Definition 1 (Common shock specification) *For all $n \in \mathcal{N}$ and $t \in \mathbb{N}$, $\theta_n(t) = \bar{\theta}_n + \varepsilon(t)$, where $\bar{\theta} \in \mathbb{R}^N$ is such that for all $n > m$, $\bar{\theta}_n > \bar{\theta}_m$, the process $\{\varepsilon(t) : t \in \mathbb{N}\}$ is i.i.d., and F denotes its c.d.f.*

In the common specification, $\bar{\theta}_n$ captures the ideological position of n relative to the other policy makers, whereas $\varepsilon(t)$ captures the shock to the policy environment in period t. Thus $(\bar{\theta}_n)_{n \in \mathcal{N}}$ allows policy preferences to vary across legislators whereas $\{\varepsilon(t) : t \in \mathbb{N}\}$ allows policy preferences to vary over time.

The legislative game

In each period t, a status quo $q(t) \in \{L, R\}$ is in place. First, $\theta(t)$ is realized and observed by all policy makers.[5] Then they choose whether to vote for the status quo or the

[4] Assumption 1 is sufficient but not necessary for our results, though it simplifies the analysis considerably. Under a proper equilibrium refinement, some degree of preference reversal would not alter our results. All we would need is that conditional on the preferences of any two legislators disagreeing, the more leftist always prefers L in expectation and the more rightist always prefers R in expectation.

[5] This assumtion is without loss of generality: we will see later that in equilibrium, each policy maker n conditions her action in period t only on $\theta_n(t)$; hence, whether she observes the preferences of the other policy makers is inconsequential.

alternative policy. If the set of policy makers who vote for latter policy is a winning coalition, this policy is implemented. Otherwise, the status quo stays in place. The implemented policy determines the payoffs for period t and becomes the status quo for the next period $q(t+1)$.

The institutions

In our stylized environment, the institutional arrangement is summarized by the set of winning coalitions needed for a policy change. We allow this set to depend on the identity of current status quo: for all $q \in \{L, R\}$, Ω_q denotes the set of winning coalitions when the status quo is q, Ω denotes (Ω_L, Ω_R), and $\Gamma(\Omega)$ denotes the game thus defined. We impose the following conditions on Ω.

Assumption 2 *For all $q \in \{L, R\}$,*
(i) Monotonicity: if $C \in \Omega_q$ and $C \subset C'$, then $C' \in \Omega_q$,
(ii) Nonemptyness: $\mathcal{N} \in \Omega_q$,
(iii) Properness: if $C \in \Omega_q$ and $q' \neq q$, then $N \setminus C \notin \Omega_{q'}$.

Conditions (i) to (iii) are standard in the literature on voting (see, e.g., Austen-Smith and Banks 1999). Monotonicity ensures that an additional vote in favor of an alternative can only increase the likelihood that this alternative is implemented. Nonemptyness means that the voting rule is Paretian. Properness requires that if a coalition can change the policy in place, those outside this coalition cannot immediately reverse this change.

Assumption 2 allows for a large class of institutions. For instance, if we set $\Omega_R = \Omega_L = \Omega^M$ where

$$\Omega^M \doteq \{C \subseteq \mathcal{N} : |C| \geq M\}, \tag{1}$$

then Ω corresponds to a supermajority with threshold M. Assumption 2 is satisfied for all $M \geq (N+1)/2$. The two polar cases $M = (N+1)/2$ and $M = N$ correspond to simple majority rule and unanimity rule, respectively.

Assumption 2 allows also for nonanonymous institutions, such as the combination of simple majority rule and a veto player (or gate keeper) $v \in \mathcal{N}$:

$$\Omega_R = \Omega_L = \left\{C \subseteq \mathcal{N} : C \in \Omega^{(N+1)/2} \text{ and } v \in C\right\}.$$

Nonanonymous institutions are the de-facto rules in many democracies in which the legislative body uses simple majority but is subject to the negative agenda control of the majority leader, or the veto power of the president.[6]

[6] In the case of a presidential veto, the veto player v is not a member of the legislature, so the voting rule is actually characterized by $\Omega_R = \Omega_L = \left\{C \subseteq \mathcal{N} : C \setminus \{v\} \in \Omega^{(N+1)/2} \text{ and } v \in C\right\}$.

Assumption 2 also allows for institutions with a policy bias, since Ω_L can differ from Ω_R. For instance, institution $\Omega = \left(\Omega^{(N+1)/2}, \Omega^M\right)$ prescribes simple majority rule to decide whether to replace status quo L by R and a qualified majority rule with threshold $M > (N+1)/2$ to decide whether to replace status quo R by L. Such institutions are used by many US states to legislate taxation: while simple majority is needed to lower taxes, a qualified majority is required to raise them (see Section 5.2).

Equilibrium

As standard for dynamic voting games, we look for Markov perfect equilibria in stage-undominated strategies (henceforth, equilibria) as defined in Baron and Kalai (1993). A Markov strategy for player n in $\Gamma(\Omega)$ maps in each period t the current state $\theta(t)$ and the current status quo $q(t)$ into a probability distribution over votes. Stage undomination requires that in each period, each player votes for the policy that gives her the greater continuation payoff. This equilibrium refinement rules out pathological equilibria such as all players always voting for the status quo. We assume that when indifferent, a player votes for R, but this assumption is made only for expositional convenience and does not affect the set of equilibria that we find.

Comments

Two comments on the modeling assumptions are in order. First, the policy space consists of two alternatives, which is clearly an abstraction, albeit a useful one. Since there is only one possible alternative to the status quo, this assumption implies that in each period, the bill under consideration is exogenous. As a result, we do not need to specify the details of the procedures that govern how bills are chosen, and can completely characterize the political decision process by the collections of winning coalitions Ω. It is also worth emphasizing, that our results do not rely on the absence of "compromise" alternatives that lie between L and R. In a setting with two players, Dziuda and Loeper (2015, Section 4.2) show that under reasonable assumptions on the bargaining procedure, similar voting distortions arise when players can chose from a one-dimensional space of alternatives, and there is no reason to suspect that the same would not hold in the setting of this paper. Allowing for more than two alternatives, however, would come at a significant cost in terms of technical and expositional complexity, and would not allow for the richness of the results obtained in this paper.[7]

Second, we consider policies that are continuing in nature: policies that do not have an expiration date and remain in effect until a new agreement is reached. Many policies have this

[7] Specifically, Dziuda and Loeper (2015, Section 4.2) consider a bargaining procedure in which the status quo is replaced by sequential considering of incremental changes. This bargaining procedure has also been studied in related environment in Gradstein (1998), Riboni and Ruge Murcia (2010), and Acemoglu et al (2014).

feature. For instance, in the U.S., this is the case of most spending and tax policies (Gale and Orszag 2003, Levit and Austin 2014). Similarly, labor laws and minimum wage legislation tends to be continuing in nature (Clinton 2011). Sunset provisions, i.e., clauses that attach an expiration date to a legislation, historically have been the exception rather than the norm (see Dziuda and Loeper 2016, or Austen-Smith et al, 2016, for further discussion on why sunset provisions are may be rare).

4 Equilibrium

4.1 The static model

Since most of the lawmaking literature is static or assumes that legislators consider each legislative period in isolation (e.g., Krehbiel 1998, Brady and Volden 2006), let us first consider as a benchmark the game $\Gamma^0(\Omega)$ in which policy makers play only the first period $t = 0$ of $\Gamma(\Omega)$. In the unique equilibrium of this static game, each policy maker n votes sincerely. That is, she votes for R if $\theta_n(0) \geq 0$ and for L if $\theta_n(0) < 0$, irrespective of the status quo. Under such behavior, Assumption 1 implies that if a policy maker n votes for R, so do all more rightist policy makers $k \geq n$, and if she votes for L, so do more leftist policy makers $k \leq n$. Therefore, if n is such that

$$\{n, ..., N\} \in \Omega_L \text{ and } \{n+1, ..., N\} \notin \Omega_L, \tag{2}$$

then status quo L is replaced by R if $\theta_n(0) \geq 0$ and it stays in place if $\theta_n(0) < 0$. Thus, when the status quo is L, the equilibrium outcome always coincides with n's vote. Conditions (i) and (ii) in Assumption 2 guarantee that (2) characterizes a unique n, which we label $l(\Omega_L)$ and refer to as *the pivot under status quo* L. The pivot under status quo R, which we denote $r(\Omega_R)$, is defined in a symmetric fashion by

$$\{1, ..., n\} \in \Omega_R \text{ and } \{1, ..., n-1\} \notin \Omega_R. \tag{3}$$

The following proposition summarizes the above findings.

Proposition 1 *The unique equilibrium of* $\Gamma^0(\Omega)$ *is sincere voting: each policy maker* $n \in \mathcal{N}$ *votes for* R *if* $\theta_n(0) \geq 0$ *and for* L *otherwise. If the status quo is* L *(*R*), policy maker* $l(\Omega_L)$ *(*$r(\Omega_R)$*) is* pivotal *in the sense that the equilibrium outcome always coincides with her vote.*

Proposition 1 implies that in a static environment, institutions have no impact on policy makers' behavior. They only affect how votes are aggregated via the identity of the two

pivots $l(\Omega_L)$ and $r(\Omega_R)$. As we will see in the subsequent analysis, this is no longer true in the dynamic game.

The pivots play a crucial role in the rest of the analysis, so it is instructive to discuss who they are as a function of the voting rule. Assumption 1 together with conditions (i) and (iii) in Assumption 2 imply that $l(\Omega_L) \leq r(\Omega_R)$, which means that the pivot under status quo L is more leftist than the pivot under status quo R. When $l(\Omega_L) = r(\Omega_R)$, we shall say that Ω is *quasi-dictatorial:* the equilibrium outcome of $\Gamma^0(\Omega)$ is the same as if the unique pivot were the dictator. Quasi-dictatorial institutions clearly include dictatorship, but also nondictatorial institutions such as simple majority rule: when N is odd, then $l(\Omega_L) = r(\Omega_R) = (N+1)/2$. When $l(\Omega_L) < r(\Omega_R)$, we shall say that Ω is *non quasi-dictatorial.* Examples of non quasi-dictatorial institutions include supermajority requirements, bicameral legislatures, or unicameral legislatures with a presidential veto.

4.2 The dynamic model

Let us now turn to the dynamic game $\Gamma(\Omega)$.

Proposition 2 *For any equilibrium of $\Gamma(\Omega)$, there exists $d = (d_1, ..., d_N) \in \mathbb{R}^N$ such that each policy maker $n \in \mathcal{N}$ votes for R if and only if $\theta_n + d_n \geq 0$. If Ω is quasi-dictatorial, then the unique equilibrium is sincere voting, i.e., $d = (0, ..., 0)$. If Ω is non quasi-dictatorial, in any equilibrium,*

(i) $d_1 < .. < d_N$,

(ii) $d_{l(\Omega_L)} < 0 < d_{r(\Omega_R)}$.

As in the static game $\Gamma^0(\Omega)$, in every period t in which the status quo is L (R), policy maker $l(\Omega_L)$ ($r(\Omega_R)$) is pivotal in the sense that for all realizations of $\theta(t)$, the equilibrium outcome coincides with her vote.

Proposition 2 has three noteworthy implications.[8] First, policy makers vote sincerely in $\Gamma(\Omega)$ only when Ω is quasi-dictatorial. When Ω is non quasi-dictatorial, they vote as if their preferences where $\theta_n(t) + d_n$ instead of $\theta_n(t)$. We call the term $\theta_n(t) + d_n$ her *strategic preferences* in period t, and d_n her *voting distortion.* Second, when Ω is non quasi-dictatorial, the voting distortions d have a polarizing effect. To see this, note that for any

[8] As mentioned in Section INSERT, this result parallers the result of Dziuda and Loeper (2016) for two players and the unanimity rule, but because of technical differences between these two papers, proving this result independently is easier than adapting the proof od Dziuda and Loeper (2016).

non quasi-dictatorial Ω, for all $n > m$,

$$(\theta_n(t) + d_n) - (\theta_m(t) + d_m) = (\theta_n(t) - \theta_m(t)) + (d_n - d_m) > \theta_n(t) - \theta_m(t) > 0, \qquad (4)$$

with probability 1, where the first inequality follows from part (i) and the last inequality follows from Assumption 1. The left-hand side of (4) can be interpreted as *strategic polarization* between policy makers n and m. Hence, under any non quasi-dictatorial rule, strategic polarization for any two policy makers n and m is larger that their sincere ideological polarization $\theta_n(t) - \theta_m(t)$.

Finally, Proposition 2 has important implications for policy dynamics. To compare policy dynamics across games, equilibria, and institutions, we use the following definition.

Definition 2 *For a given institution Ω and strategy profile σ, we say that gridlock occurs in state $\theta \in \mathbb{R}^N$ if the actions prescribed by σ are such that whenever $\theta(t) = \theta$, then $q(t)$ stays in place.*

From Propositions 1 and 2, gridlock occurs in state θ when the pivots disagree in that state. In $\Gamma^0(\Omega)$ the latter happens when

$$\theta_{l(\Omega_L)} < 0 \leq \theta_{r(\Omega_R)},$$

and in $\Gamma(\Omega)$ when

$$\theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \leq \theta_{r(\Omega)} + d_{r(\Omega)}.$$

Proposition 2 part (ii) therefore implies that the set of gridlock states is greater in $\Gamma(\Omega)$ than in $\Gamma^0(\Omega)$ in the inclusion sense. Thus, the strategic polarization arising in the dynamic game exacerbates gridlock: policy change occurs less frequently than predicted by the static model.[9,10]

The intuition for Proposition 2 is as follows. In $\Gamma(\Omega)$, policy makers' votes determine not only today's policy, as in the static game $\Gamma^0(\Omega)$, but also tomorrow's status quo. The identity

[9] Our definition of gridlock is a natural extension of the canonical definition of Krehbiel (1998) adapted to the dynamic model. Krehbiel (1998) defines gridlock as a set of policy positions that are not dominated by any other policy. In our setting, no policy satisfies this by assumption for all states, but one can specify the set of states for which policy makers are gridlocked on a particular policy x. We define gridlock as the set of states for which policy makers are gridlocked on every policy.

[10] Part (iii) of Assumption 2 excludes decision rules in which a coalition C of policy makers can change the policy, and another coalition C' of policy makers disjoint from C can reverse this policy change (e.g., a minority rule). Such rules are rarely used in practice, and in our model, they would lead to multiple policy changes within a single period because under such rules, $l(\Omega_L) > r(\Omega_R)$. Nevertheless, it is interesting to note that if we allowed for such rules but restricted legislators to only one policy change per period, our results would be overturned. Legislators would behave in a more moderate way and gridlock would decrease. We thank Salvador Barbera for pointing this out to us.

of tomorrow's status quo matters only in gridlock states. Hence, if policy makers expect some gridlock to occur in the future, then they have incentives to distort their current votes in favor of the alternative that they prefer conditional on gridlock. Since more rightist policy makers have stronger preferences for R over L, they will have (weakly) larger distortions, which implies that the ideological order from Assumption 1 is preserved in votes. As a result, the same players as in $\Gamma^0(\Omega)$ are pivotal. Hence, if Ω is quasi-dictatorial, then $l(\Omega_L) = r(\Omega_R)$. In this case, there is no gridlock, and therefore the policy makers have no incentive to distort their voting behavior. This explains why $d = (0, ..., 0)$ is the unique equilibrium. If Ω is non quasi-dictatorial, then $l(\Omega_L) < r(\Omega_R)$, so in all gridlock states $l(\Omega_L)$ prefers L and $r(\Omega_R)$ prefers R. This explains part (ii). The strict inequalities from part (i) then follow from the fact that the probability of gridlock is strictly positive.

The intuition above explains also why voting distortions take the following form.

Corollary 1 *For any equilibrium d of $\Gamma(\Omega)$, let $G = \left\{\theta : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \leq \theta_{r(\Omega)} + d_{r(\Omega)}\right\}$ and $\bar{\theta}_n = \int \theta_n dP(\theta)$ Then for all $n \in \mathcal{N}$,*

$$d_n = \frac{\delta \int_{\theta \in G} \theta_n dP(\theta)}{1 - \delta P(G)}, \tag{5}$$

$$\frac{1}{N} \sum_{n \in \mathcal{N}} d_n = \frac{\delta \frac{1}{N} \sum_{n \in \mathcal{N}} \int_{\theta \in G} \theta_n dP(\theta)}{1 - \delta P(G)}. \tag{6}$$

Equation (6) says that strategic polarization is not the only distortion that may occur. On average, policy makers' votes may be more rightist or leftist than their ideologies.

Proposition 2 and Corollary 1 imply that there is a fundamental difference between the insights from the static and the dynamic models. At first sight, the predictions of these models seem observationally equivalent. In both models, each legislator votes for the policy that is closer to her ideal point in a given state, the only difference being that in the static theory, this ideal point reflects her intrinsic preference $\theta_n(t)$, whereas in the dynamic theory the ideal policy is also affected by strategic, forward looking considerations, $\theta_n(t) + d_n$. However, this apparent equivalence overlooks an important point. First and foremost, the static approach treats policy makers' preferences as an independent primitive. In contrast, Proposition 2 implies that these preferences are equilibrium objects, and as such, are endogenous to the political decision process Ω. Hence, when analyzing the impact of institutional changes, one cannot take as given the preferences induced from the current legislative behavior, but has to understand how these strategic preferences will change with Ω. Second, policy makers' strategic preferences depend crucially on their expectations about the future (equation 5). Hence, changes in these expectations, even when not accompanied

by any observable changes in the current institution or environment, are likely to affect current behavior.

In the subsequent sections we analyze in more detail how policy makers' behavior depends on the political institution (Section 5) and on policy makers' expectations about future economic and political environment (Section 6).

Before we move on, let us note that when Ω is non quasi-dictatorial, $\Gamma(\Omega)$ may have multiple equilibria, which makes the comparative statics problematic. In the appendix (see Corollary 2), however, we prove that there exists an equilibrium in which the degree of strategic polarization between any two policy makers is smallest. The comparative statics in the following sections are derived for this equilibrium, which we denote by $d(\Omega)$.[11]

5 Institutions and voting distortions

5.1 Non quasi-dictatorial institutions and strategic polarization

In this section we investigate in more detail how the equilibrium voting behavior of policy makers depends on the details of the voting institution. To this end, we use the following partial order on institutions.

Definition 3 *Let Ω and Ω' be two institutions. We say that Ω' is requires a greater consensus than Ω if $\Omega'_L \subseteq \Omega_L$ and $\Omega'_R \subseteq \Omega_R$.*

In words, Ω' requires a greater consensus than Ω if changing the status quo requires the approval of a greater set of policy makers under Ω' than under Ω. That is, any winning coalition under Ω' is also a winning coalition under Ω, but not necessarily the reverse. In the case of supermajoritarian institutions, rules that with a higher supermajority threshold require a greater consensus. Likewise, adding a veto player increases the degree of consensus required. Assumption 1 together with the definition of $l(\Omega_L)$ and $r(\Omega_R)$ imply that an institution that requires greater consensus leads to more extreme pivots. Formally, if Ω' requires a greater consensus than Ω, then

$$l(\Omega_L^{'}) \leq l(\Omega_L) \leq r(\Omega_R) \leq r(\Omega_R^{'}).^{12} \tag{7}$$

[11] Using arguments similar to those in Dziuda and Loeper (2016), one can show that the equilibrium with the smallest degree of strategic polarization is also Pareto best and minimizes in the inclusion sense the set of gridlock states across equilibria. It is also worth noting that the comparative statics derived in the rest of the paper holds as well for the equilibrium with the largest degree of polarization.

[12] To see this, note that by definition of $l(\Omega_L)$, $\{l(\Omega_L)+1,...,N\} \notin \Omega_L$, and since $\Omega'_L \subseteq \Omega_L$, $\{l(\Omega'_L)+1,...,N\} \notin \Omega'_L$, so $l(\Omega'_L) \leq l(\Omega_L)$.

The following proposition shows that the degree of strategic polarization increases monotonically with the degree of consensus required by the institution.

Proposition 3 *If Ω' requires a greater consensus than Ω, then for all $n, m \in \mathcal{N}$ such that $n > m$, and all t,*

$$0 \leq \theta_n(t) + d_n(\Omega) - (\theta_m(t) + d_m(\Omega)) \leq \theta_n(t) + d_n(\Omega') - (\theta_m(t) + d_m(\Omega')).$$

If Ω is non quasi-dictatorial, if $l(\Omega_L) \neq l(\Omega'_L)$ or $r(\Omega_R) \neq r(\Omega'_R)$, and if for each n, the distribution of θ_n has full support,[13] *then all inequalities above are strict.*

The intuition for Proposition 3 is as follows. From (7), as the degree of consensus required to change the policy increases, ideologically more extreme policy makers become pivots, and hence the probability that pivots disagree in the future goes up. The increased probability of future disagreement increases the incentives of each policy maker to defend her preferred policy conditional on gridlock, and clearly legislators with stronger preferences for R over L respond more strongly to that increase in future gridlock probability. This intuition implies also the following.

Proposition 4 *If Ω' requires greater consensus than Ω, then*

$$d_{l(\Omega'_L)}(\Omega') \leq d_{l(\Omega'_L)}(\Omega) \leq d_{l(\Omega_L)}(\Omega) \leq 0 \leq d_{r(\Omega_R)}(\Omega) \leq d_{r(\Omega'_R)}(\Omega) \leq d_{r(\Omega'_R)}(\Omega'). \tag{8}$$

If Ω is non quasi-dictatorial, if $l(\Omega_L) \neq l(\Omega'_L)$ or $r(\Omega_R) \neq r(\Omega'_R)$, and if for each n, the distribution of θ_n has full support, then all inequalities in (8) are strict.

Proposition 4 states that as the degree of consensus required by the institutions increases, the degree of strategic polarization between the pivots increases for two reasons. First, from (7), more extreme decision makers become pivots, and from Proposition 2, for a fixed institution Ω, more extreme policy makers are more strategically polarized. This effect explains the four inner inequalities in (8). Second, the degree of strategic polarization between any two given voters increases, which explains the two outer inequalities in (8).

An important consequence of Proposition 4 is that the static and the dynamic approaches lead to different conclusions on the impact of institutional change, and this difference comes precisely from the aforementioned second effect. To see why, note that when assessing the impact of an institutional change from Ω to Ω', the static approach assumes that policy makers' past voting behavior under Ω, as given by their strategic preferences $(\theta_n(t) + d_n(\Omega))_{n \in \mathcal{N}}$,

[13] By full support, we mean that for all $a < b$, the probability that $\theta_n \in (a, b)$ is strictly positive.

is sincere, and thus that they would keep on voting according to this revealed ideology $(\theta_n(t) + d_n(\Omega))_{n\in\mathcal{N}}$ after implementing Ω'. Given the new pivots $l(\Omega')$ and $r(\Omega')$, the static approach would then conclude that gridlock under Ω', as defined in Definition 2, will occur when

$$\theta_{l(\Omega')}(t) + d_{l(\Omega')}(\Omega) \leq 0 < \theta_{r(\Omega')}(t) + d_{r(\Omega')}(\Omega).$$

The dynamic approach implies instead that the set of gridlock states under Ω' will be

$$\theta_{l(\Omega')}(t) + d_{l(\Omega')}(\Omega') \leq 0 < \theta_{r(\Omega')}(t) + d_{r(\Omega')}(\Omega').$$

The two outer inequalities in (8) imply that the static approach underestimates the inertial effect of replacing Ω by Ω'. Specifically, it takes into account the fact that the new pivots are farther away from each other on the ideological spectrum, but it does not take into account the fact these new pivots are less likely to agree under Ω' than under Ω.

These results provide potentially testable implications. Consider an institutional reform which, say, increases the degree of consensus required for policy change. Then any two policy makers who are in office before and after the reform takes place should exhibit a greater degree of polarization in their voting behavior after the reform is implemented.

Proposition 3 suggests also an alternative reading of the relationship between legislative polarization and the use of the filibuster in the US Congress. Most scholars agree that legislators' polarization, as inferred from their voting behavior, has grown dramatically since the 70s (see, e.g, McCarty et al. 2008). During the same period, the filibuster—once an infrequently used tool reserved for the most important and controversial bills—has become a routine practice in American politics (see Table 3, and also Koger 2010, and chapter 5 of Binder and Smith 1997). Political scientists typically interpret the greater polarization in legislators' vote as an increase in their ideological polarization, which caused the more frequent use of the filibuster (Krebhiel 1998, McCarty 2007). Proposition 3 suggests a possible reverse causality: the change in institutional practice increased the de-facto required degree of consensus.

5.2 Biased institutions and average voting distortions

Some democratic institutions use different voting rules depending on the policy change under consideration. For example, in 16 U.S. states, a tax increase requires the approval of a qualified majority in each house, whereas a tax cut can be approved by a simple majority. Similarly, in the U.S. budget process, the Byrd Rule essentially requires a filibuster-proof majority to pass bills that raise the deficit, whereas a simple majority is sufficient to pass

bills that lower it.[14] The explicit goal of these nonneutral voting rules was to limit the growth of the public sector and of the public debt. Indeed, within a static framework, increasing a hurdle to pass for a policy change in one direction, decreases the probability that such policy change occurs. However, the analysis below shows that once the strategic effects highlighted in this paper are taken into account, a fiscally conservative voting rule can induce a more liberal voting behavior on average, and can even lead to more liberal policies. This insight is consistent with the empirical finding that the effect of such biased voting rule on the level of state taxes in the US has been quite modest, if any.[15]

Definition 4 *An institution Ω is biased in favor of policy R if $\Omega_L \subseteq \Omega_R$ and $l(\Omega) + r(\Omega) > N$.*

In words, an institution is biased in favor of R if implementing R under status quo L requires a strictly smaller consensus than implementing L under status quo R.

Proposition 5 below identifies an environment in which the legislature as a whole has no ideological bias, but when operating under a rule biased in favor of R, the legislature votes as if it was in favor of L on average. In other words, the policy makers anticipate that it may be hard to replace R with L in the future, and as a result, they are more inclined to vote for L today.

Proposition 5 *Consider the common shock specification in Definition 1 where $\left(\bar{\theta}_n\right)_{n\in\mathcal{N}}$ and $\varepsilon(t)$ are symmetrically distributed around 0. If Ω is biased in favor of R, then the average voting distortion is biased in favor of L, that is, $\frac{1}{N}\sum_n d_n < 0$.*

Proposition 5 raises a question whether voting distortions in favor of L may be strong enough to undo the bias of a biased institution. The example below shows that the answer is affirmative.

Example 1 *Consider the common shock specification in Definition 1, where $\varepsilon(t)$ follows a standard normal distribution. Unlike in Proposition 5, however, assume that the legislature*

[14] The U.S. federal budget process is governed by the Congressional Budget Act of 1974, which allows legislators to bypass the filibuster via the reconciliation process. This act was amended in 1985 and 1990 by the Byrd Rule to restrict the use of reconciliation (and therefore restore the use of the filibuster) against provisions that increase the deficit beyond the years covered by the reconciliation measure.

[15] For the period 1980-2008, in the average state with no supermajority requirement, taxes as a share of personal income have been between 9.7 percent and 10.9 percent, while in the seven states with supermajority requirements, they have been between 9.7 percent to 10.8 percent (Leachman, Johnson, Grundman, 2012, and Jordan and Hoffman, 2009). Knight (2000) reports that in 1995, among the continental states, states with supermajoritarian requirements had identical average effective tax rates of 7.13% as states without such requirements. Using fixed effects models, Knight (2000) and Besley and Case (2003) find that supermajoritarian requirements reduce taxes by only about $50 per capita.

is ideologically biased toward L, with the median policy maker's ideological position being $\bar{\theta}_{(N+1)/2} = -0.5$. Under the simple majority rule $\left(\Omega^{(N+1)/2}, \Omega^{(N+1)/2}\right)$, the median voter is always pivotal, and there are no voting distortions; hence, the probability that L is implemented in any period is $\Pr\left(-0.5 + \varepsilon < 0\right) = \Phi\left(0.5\right) = 0.7$. Suppose now that we increase the hurdle needed to implement L. The median policy maker remains pivotal for the policy change from L to R,whereas a more rightist policy maker becomes the pivot under R, $\bar{\theta}_r > -0.5$. Hence, the probability that R is replaced by L indeed goes down, and is equal to $\Phi\left(-\bar{\theta}_r - d_r\right) < \Phi\left(-\bar{\theta}_r\right) < \Phi\left(0.5\right)$. However, since the median policy maker distorts her voting behavior in favor of L, $d_{(N+1)/2} < 0$, the probability that L is replaced by R also goes down, $1 - \Phi\left(-\bar{\theta}_{(N+1)/2} - d_{(N+1)/2}\right) < 1 - \Phi\left(0.5\right)$. Hence, in terms of taxes, making a rule biased against tax increases, makes both tax increases and decreases less likely. The latter is more likely to dominate if the median policy maker has stronger preferences than $r\left(\Omega_R\right)$.

Figure 2 below depicts the invariant probability of policy L; i.e., the probability that in a distant future the policy is L, as a function of $\bar{\theta}_{Rl}$ for the biased voting rule (solid line) and for the simple majority (dashed line). We see that increasing the bias in favor of R, which is equivalent to increasing $\bar{\theta}_r$, does not monotonically increase the long-run prevalence of R, and it can even make L more likely to be implemented in the long-run.[16]

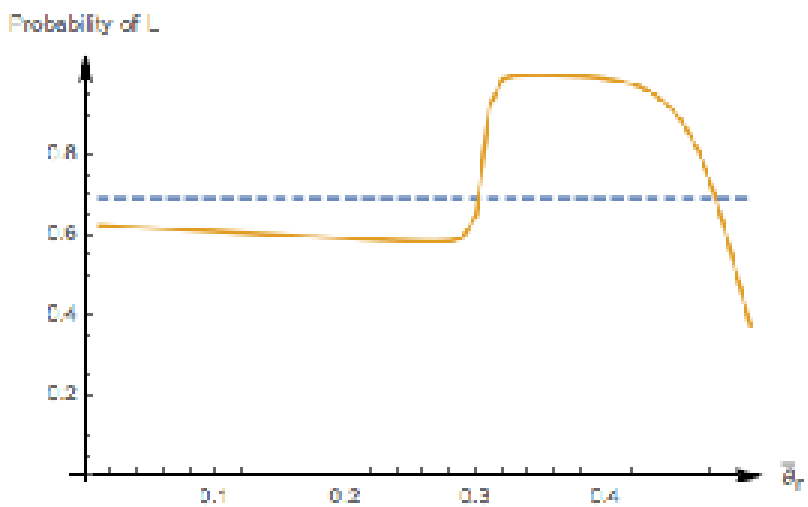

Figure 2: Incidence of L as a function of the bias of the voting rule in favor of R.

[16]Increasing the bias in favor of R initially decreases the probability of L because the ideological polarization of the pivots is relatively low; hence, their voting distortions are low as well. For $\bar{\theta}_M \approx 0.3$, the least strategically polarized equilibrium changes discontinuously with $\bar{\theta}_M$, and the strategic distortion of the median policy maker in favor of L is large enough to overcome the bias of the voting rule in favor of R. As $\bar{\theta}_M$ increases even further, the rightist pivot's preferences for R become stronger and dominate the strategic distortion of the median policy maker in favor of L.

6 The Role of Expectations

We have concluded Section 4.2 by recognizing that expectations about the future play a crucial role in determining the policy makers' current behavior. In this section, we first look at how policy makers' voting behavior depends on whether they expect their preferences over the policies to be relatively stable. Second, we allow the policy makers to have expectations over the identity of the future pivots.

6.1 Volatility of the environment

The state θ can be interpreted as the current economic situation, state of knowledge on the issue voted upon, or voters' current sentiment. In some areas, the state is likely to change more frequently than in others. Hence, the process $\{\theta(t) : t \in \mathbb{N}\}$ may vary across issues. In this section, we investigate how strategic polarization and gridlock depend on policy maker's expectations about the frequency with which the state changes.

To this end, we relax the assumption that the state is i.i.d. over time. Instead, we assume that in each period $t \geq 1$, conditional on $\theta(t-1)$, $\theta(t) = \theta(t-1)$ with probability $1-v$, and with probability v, $\theta(t)$ is redrawn independently of the previous states of nature from a probability distribution P which is independent of t. Thus, P captures the shocks to the environment, and $v \in [0,1)$ measures the volatility of the environment.

We have established so far that non quasi-dictatorial institutions increase gridlock over policies. Such gridlock is bound to be particularly detrimental in environments in which frequent policy changes are a desirable response to a volatile environment. In the next proposition we show that unfortunately, it is in such environments that strategic polarization, and hence gridlock, tend to be greater.

Proposition 6 *Let Ω be an inclusive voting rule, let $v, v' \in [0,1)$ be such that $v < v'$, and let d and d' be the profiles of equilibrium distortions corresponding to v and v' respectively. Then*

$$d'_{l(\Omega_L)} < d_{l(\Omega_L)} < 0 < d_{r(\Omega_R)} < d'_{r(\Omega_R)}.$$

When the environment is static, that is, when $v = 0$, then $d = (0, .., 0)$.

The intuition for Proposition 6 is as follows. As the volatility of the environment increases, the policy implemented today is more likely to require a revision tomorrow; hence, securing a favorable status quo becomes more important relative to implementing the desirable policy today.

Proposition 6 suggests that the impact of the volatility of the environment on the volatility of the policy is ambiguous. As the environment becomes more volatile, more frequent

shocks prompt more frequent policy changes, but policy makers become strategically more polarized, and are thus less likely to agree on a policy response after a shock. Figure 3 below demonstrates that the latter effect may dominate, and policy persistence may be greater in environments that require more frequent policy reforms.

Figure 3 considers again the common shock specification from Definition 1, and supposes that under Ω, the pivots are symmetric, i.e., $\bar{\theta}_{r(\Omega_R)} = -\bar{\theta}_{l(\Omega_L)} = 0.5$. For this environment, figure 3 depicts policy volatility—measured by the probability that the policy changes in the next period—as a function of the volatility of the environment v. Initially, these two volatilities move together. This is because for low volatility, the strategic effect is small. However, as volatility becomes large, the strategic effect dominates and the policy becomes more persistent.

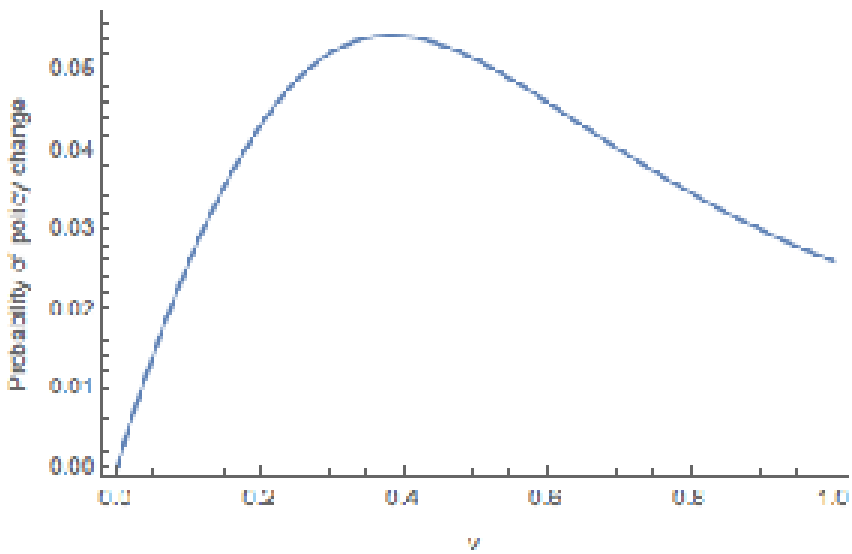

Figure 3: Probability of policy change as a function of preference volatility.

Tax, welfare, or trade are prominent examples of volatile policy domains. The goal of most tax reforms oscillate between stimulating the economy (i.e., lowering tax rates) and reducing the deficit (i.e., increasing tax rates). These reforms are prompted by recurring shocks such as business cycles, changes in fiscal needs, or the vagaries of public opinion. Likewise, since WWII, welfare policies have had to adapt to the emergence of new social risks, growing inequalities, and changing demographics (Hacker 2004). Trade policies must also react to sectorial shocks, domestic pressures, and the international environment. Proposition 6 provides a rationale for procedural rules which limit the degree of consensus required in

such areas. Examples of such rules include the reconciliation process, which limits the use of the filibuster for fiscal policies, or the fast track authority, which limits the ability of Congress to veto the decisions of the president for trade policies (Koger 2010).

6.2 Political turn-over and electoral expectations

The basic model assumes that policy makers stay in office indefinitely, and thus that the identity of the two pivots remains unchanged throughout the game. In reality, elections imply political turnover. In the presence of political turnover, voting biases are still given by policy makers' expected preferences in the next period conditional on future disagreement, but the future disagreement is determined by the identity of the future pivots. Legislators are likely to form expectations over who the future pivots will be, and these expectations may vary with which legislators are up for reelections, who their challengers are, whether midterm or presidential elections are on the horizon, and the latest opinion polls. In this section, we investigate formally how legislators' expectations about future political turnover shape their voting behavior.

To do so, we extent the model as follows. Let $\mathcal{N} = \{1, ..., N\}$ denote the set of political positions (e.g., presidency, congressional seats) that have to be filled, and $\mathcal{C} = \{1, ..., C\}$ the set of potential candidates for these positions. For all $t \in \mathbb{N}$, $e(t) \in \mathcal{C}^{\mathcal{N}}$ denotes the electoral outcome in period t : $e(t)$ is a vector describing which candidates are elected in period t. As in the basic model, the preferences of each candidate $c \in \mathcal{C}$ are given by an i.i.d. process $\{\theta_c(t) : t \in \mathbb{N}\}$. In line with Assumption 1, we assume that for all $c, c' \in \mathcal{N}$, if $c \leq c'$, then for all t, $\theta_c(t) \leq \theta_{c'}(t)$ almost surely. To introduce electoral uncertainty, we assume that the electoral outcome $\{e(t) : t \in \mathbb{N}\}$ is a stochastic process. To allow policy makers' expectations to vary across periods, we assume that in each period t, before casting their votes, all policy makers observe a signal $s(t)$ drawn from a finite set S that is correlated with the outcome of the next elections $e(t+1)$. To retain the stationarity of the model, we assume that $\{(s(t), e(t+1)) : t \in \mathbb{N}\}$ is i.i.d. and independent of $\{\theta(t) : t \in \mathbb{N}\}$. That is, $s(t)$ gives information only about the elections in period $t+1$, and electoral outcomes are independent of the policy preferences.

To keep this extension in line with the basic model, we assume that each candidate $c \in \mathcal{C}$ cares about the implemented policy independent of whether she is in office. We acknowledge that this assumption may not always hold, but the results are likely to hold under alternative assumptions.[17]

[17] If candidates care about policies only when in office, then their patience δ decreases to reflect the probablity of electoral defeat. Conditionally on being reelected, however, a policy maker may still face uncertainty over who fills the remaining positions, and hence, the identity of the pivots.

Proposition 7 *For any equilibrium, there exists a profile of functions* $d = (d_1, ..., d_C) \in \left(\mathbb{R}^S\right)^C$ *such that for all* $t \in \mathbb{N}$, *each candidate* c *elected in period* t *votes for* R *if and only if* $\theta_c(t) + d_c(s(t)) \geq 0$. *For all possible realizations* s *of the signal,* $d_1(s) < .. < d_C(s)$.

Proposition 7 is a natural extension of Proposition 2, but reveals an interesting point overlooked by the latter. First, the voting distortions do not depend on which candidates are currently in office, $e(t)$. Instead, they depend on policy makers' current expectations about who will be in office tomorrow, $s(t)$. This is because policy makers' distort their votes in order to influence which alternative prevails in case of *future* gridlock, and the latter does not depend on who is currently in office. Note, however, that the current electoral outcome $e(t)$ still affects the policy outcome in period t, because it affects the identity of the current pivots.

To illustrate the applicability of this extension, consider a simple case in which candidates can be of only two types, say Republican or Democrat. In this case, an election can lead to three possible configurations: either both pivots are Democrats, or both pivots are Republicans, or the left pivot is Democrat while the right pivot is Republican. In the first two cases we can say that the government is united, while the last case represents a divided government.

Proposition 8 *Let* D *denote the set of realizations of* $e(t)$ *such that the government is divided, i.e., no winning coalition of elected candidates is of the same type. Then in any equilibrium, there exists* $d_D < 0 < d_R$ *such that for all* c, $d_c(s) = d_L \Pr(e(t+1) \in D | s(t) = s)$ *if* c *is a Democrat and* $d_c(s) = d_R \Pr(e(t+1) \in D | s(t) = s)$ *if* c *is a Republican.*

Thus, the voting distortions depend not on whether the current government is divided, but how likely it is to be divided in the future. When the government is united, the status quo does not matter as the government does not face a veto over its actions. Hence, if current policy makers expect a united government in the future, then they do not care about the future status quo. The status quo will matter, however, if the future government is divided, leading to large distortions in the present votes.

A careful analysis of the differences between united and divided governments in a dynamic setting is beyond the scope of this paper. It is worthwhile, however, to note that Proposition 8 implies policy distortions even under united government. To see that, consider a united Republican government that inherited high tax rates from the previous legislature. Suppose further that high public debt makes leaving those rates unchanged statically preferable for both parties. Nevertheless, Republicans may decrease the tax rates, in order to assure that they face a favorable status quo should they have to negotiate future tax rates with Democrats.

Let us investigate now the policy makers' behavior when the future government is likely to be divided in the sense that policy makers' expect two distinct future pivots, but their expectations over where these pivots are located on the ideological spectrum vary. For simplicity, we assume that policy makers have degenerate expectations about the identity of the next period pivots. That is, for any signal realization s, policy makers know with probability 1 who the next pivots $l(s)$ and $r(s)$ are. Clearly, for any equilibrium, the voting distortions $d(.)$ depend on s only through $l(s)$ and $r(s)$. Thus, they can be expressed as a function of l and r only. The following proposition characterizes $d(l,r)$.

Proposition 9 *For all c such that $r_2 < r_1 \leq c$ or $c \leq r_1 < r_2$, we have $d_c(l,r_1) \geq d_c(l,r_2)$. Likewise, for all c such that such that $l_2 < l_1 \leq c$ or $c \leq l_1 < l_2$, we have $d_c(l_1,r) \leq d_c(l_2,r)$.*

Proposition 9 states that the bias of a given policy maker c is more rightist, the closer she expects to be to the next right pivot and the farther she expects to be from the next left pivot. The intuition for this is as follows. Each legislator prefers that the pivotal legislator in the future has preferences more similar to her, as then the policy change is more in line with what a given legislator wants. Since r (l) is pivotal only when the status quo is R (L), a legislator will distort her votes in favor of R to a higher degree the more similar she is to r relative to l.

Proposition 9, like all our previous results, is derived in a stylized model; hence one has to exercise caution when applying its conclusion to the real world. Nevertheless, we will use Proposition 9 to discuss two applications, in order to demonstrate again the degree to which the lessons drawn from the dynamic model differ from those implied by the static analysis.

Consider a legislature during the first two years of a Republican presidency. Due to the coattail effect, the current legislators may expect both pivots to move (weakly) to the left after the midterm elections. That means that almost all moderate legislators, including the current left pivots, expect to be further away from the left pivot and closer to the right pivot than they currently are. Proposition 9 hence implies that the moderate legislators, including the left pivot, are becoming more polarized in favor of R. Hence, a legislature under a Republican president may be more successful in implementing conservative policies not only because of the ideological bias of the current president and current legislature, but also because of the expectation of a liberal shift in the future. The behavior of the current right pivot, however, is ambiguous. Depending on the details of the environment, she may become more or less biased in favor of R, making it easier or harder for legislature to pass liberal policy changes before the midterm elections.

And finally, Proposition 9 suggests that the effects of the 1986 Supreme Court decisions on redistricting may be more nuanced than predicted by the static literature. Theoreti-

cally, redistricting increases the ideological polarization of a particular state's delegation. If redistricting happens in an extremely conservative state, the right shift of some of its representatives does not affect the identity of the pivots, but the left shift may move the right pivot or even both pivots to the left. Proposition 9 implies then that moderate policy makers may become more rightist in their voting behavior.

7 Discussion and Conclusions

Our findings could be useful for constitutional design. As we discussed in Section 1, most modern democracies have a system of check and balances. Admittedly, these checks and balances are not designed to smooth the decision process. Rather, their role is to limit agency costs and abuses of power by any government branch. Our model shows, however, that when checks and balances are introduced in a decision process, they tend to make policy makers more polarized, which can greatly exacerbate their inherently inertial effect.

Hence, a system of checks and balances should be complemented with solutions that mitigate polarization. Such solutions are particularly important in policy domains in which the current agreement serves as a default for future negotiations—as only then voting distortions occur—and in policy domains in which fairly frequent response to exogenous shocks is desirable. One option is to allow or require a more frequent use of sunsets, especially during volatile times. Sunset provisions are clauses attached to a legislation that determine its expiration date. Thus, sunsets allow policy makers to change the policy without changing the default policy. As such, they break the dynamic linkage between today's decision and tomorrow's pivotality, and hence mitigate the voting distortions identified in this paper. Another option is to use automatic indexation rules that tie the default to some verifiable variable that is correlated with the state of nature. We leave these questions for future research.

8 Tables

State	Year Adopted	Vote Required	Mjor Features of the requirement
Arizona	1992	2/3	All tax increases
Arkansas	1934	3/4	All tax increases except related to sales and alcohol
California	1979	2/3	All tax increases
Colorado	1992	2/3	All tax increases. Tax increases automatically sunset
Delaware	1980	3/5	All tax increases
Florida	1971	3/5	Increases in the corporate income tax only
Kentucky	2000	3/5	All tax increases
Louisiana	1966	2/3	All tax increases
Michigan	1994	3/4	Increases in state property tax only
Mississippi	1970	3/5	All tax increases
Missouri	1996	2/3	All tax increases.
Nevada	1996	2/3	All tax increases
Oklahoma	1992	3/4	All tax increases
Oregon	1996	3/5	All tax increases
South Dakota	1996	2/3	All tax increases
Washington	1993	2/3	All tax increases

Table 1: Rules governing tax increases. Source: National Conference of State Legislators (http://www.ncsl.org/issues-research/budget/state-tax-and-expenditure-limits-2010.aspx)

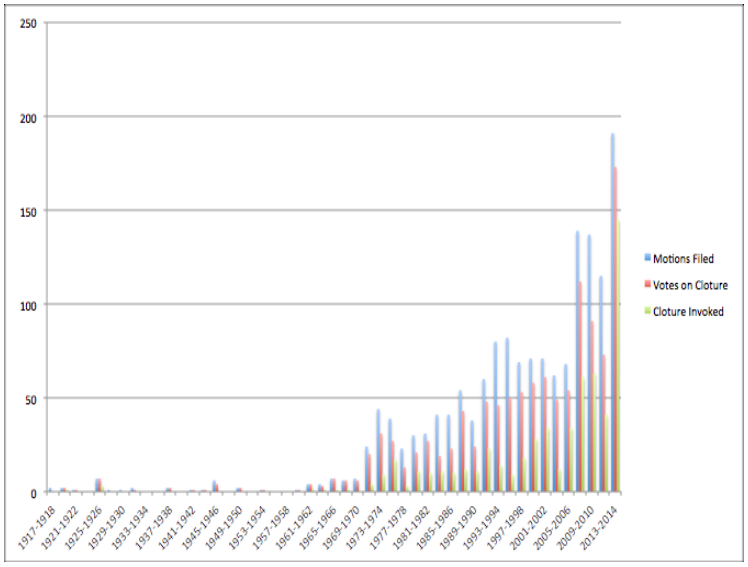

Table 2: The prevalence of the filibuster in the U.S. Senate. Source: U.S. Senate

9 Appendix

All results of this paper are proved for the case in which the state $\theta(t)$ is redrawn only with probability v.

Throughout the appendix, we use the following notations:

Notation 1 *For all voting rules Ω, all $v \in [0,1]$, and all $d \in \mathbb{R}^N$,*

$$\begin{aligned} \Theta(\Omega, d) &\equiv \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} + d_{r(\Omega_R)} \geq 0\right\}, \\ H_n((1-v), \Omega, d) &\equiv \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega,d)} (\theta_n + d_n)\, dP(\theta). \end{aligned} \tag{9}$$

We omit $(1-v)$ and Ω in the argument of l, r, Θ and H when these parameters are clear from the context.

Proof of Proposition 1. The proof is straightforward and hence omitted. ■

Lemma 1 *A strategy profile σ is an equilibrium of the dynamic game $\Gamma(\Omega)$ if and only if there exists $d \in \mathbb{R}^N$ such that $d = H(v, \Omega, d)$ and each legislator $n \in \mathcal{N}$ votes for R in period t if and only if $\theta_n(t) + d_n \geq 0$. Moreover, at any such equilibrium, $d_1 \leq ... \leq d_N$.*

Proof. Let σ be an equilibrium. Since σ is Markov, in every t, it maps $(\theta(t), q(t))$ into a distribution over votes $\{L, R\}$. Let $V_n^\sigma(\theta, q)$ be expected continuation value of policy maker n from period $(t+1)$ onwards, conditional on $\theta(t) = \theta$, on $q(t+1) = q$, and on continuation play σ. Stage undomination (together with our tie-breaking rule) requires that in t, σ prescribes n to vote for R if and only if $\theta_n(t) + \delta V_n^\sigma(\theta(t), R) \geq \delta V_n^\sigma(\theta(t), L)$. Since this inequality does not depend on $q(t)$, we conclude that the actions prescribed by σ depend on the current state but not on the current status quo. This immediately implies that in σ players revise the policy only when $\theta(t)$ is redrawn.

For all $q \in \{L, R\}$, let $W_n^\sigma(q)$ be the expected continuation value of the game for player n from period $(t+1)$ onwards, conditional on on $\theta(t+1)$ being redrawn (but not on $\theta(t+1)$'s realization), on $q(t+1) = q$, and on continuation play σ. Note that since σ is Markov, and since $\theta(t+1)$ is redrawn, $W_n^\sigma(q)$ does not depend on the realization of previous states. The difference in the expected stream of payoffs for player n from implementing policy R versus L in period t can then be expressed as follows:

$$\begin{aligned} & \theta_n(t) + \sum_{k=1}^{\infty} \delta^k (1-v)^{k-1} \left((1-v)\theta_n(t) + vW_n^\sigma(R)\right) - \sum_{k=1}^{\infty} \delta^k (1-v)^{k-1} \left((1-v) \times 0 + vW_n^\sigma(L)\right) \\ = \; & \frac{1}{1-\delta(1-v)}\theta_n(t) + \frac{\delta v}{1-\delta(1-v)} \left(W_n^\sigma(R) - W_n^\sigma(L)\right). \end{aligned} \tag{10}$$

In period t, if (10) is nonnegative (negative), then R (L) is the unique stage-undominated action for player n. Hence from (10), σ_n must prescribe n to vote for R if and only if

$$\theta_n(t) \geq \delta v \left(W_n^\sigma(L) - W_n^\sigma(R)\right) \doteq -d_n^\sigma. \tag{11}$$

Therefore, for any equilibrium σ, there exists $d^\sigma \in \mathbb{R}^n$ such that n votes for R if and only if $\theta_n(t) + d_n^\sigma \geq 0$, and d_n^σ is given by (11).[18]

Part (iii) of Assumption 2 together with the already established fact that σ does not depend on the current status quo imply that that the state space can be grouped into three sets: a set of states in which R is implemented irrespective of the status quo, a set of states in which L is implemented irrespective of the status quo, and a set of states in which either status quo stays in place. Let Θ^σ denote the last set. Note that $q(t+1)$ affects the policy outcome in $t+1$ only when $\theta(t+1) \in \Theta^\sigma$, and for all $\theta(t+1) \in \Theta^\sigma$, the status quo stays

[18] Technically, the strategy does not have to specify the behavior for θ that are not in the support of $P(\theta)$, so if the support of θ is not connected, the thresholds may not be uniquely determined. We ignore this issue for simplicity, as the resulting multiplicity does not reverse any of our findings but only complicates the exposition.

in place. Hence, $W_n^\sigma(R) - W_n^\sigma(L)$ is simply the expectation of (10) over all $\theta \in \Theta^\sigma$, so

$$W_n^\sigma(R) - W_n^\sigma(L) = \int_{\theta\in\Theta^\sigma} \left[\frac{1}{1-\delta(1-v)}\theta_n + \frac{\delta v}{1-\delta(1-v)}\left(W_n^\sigma(R) - W_n^\sigma(L)\right)\right] dP(\theta). \tag{12}$$

Isolating $W_n^\sigma(R) - W_n^\sigma(L)$ from (12) and plugging it into (11), we obtain $d_n^\sigma = -\frac{\delta v \int_{\theta\in\Theta^\sigma}\theta_n dP(\theta)}{1-\delta(1-v)-\delta v P(\Theta^\sigma)}$. Using Assumption 1, we can then conclude that $d_1^\sigma \leq ... \leq d_N^\sigma$.

Since players vote as if they were playing $\Gamma^0(\Omega)$ but their current preferences were given by $\theta_n + d_n^\sigma$ instead of θ_n, Assumption 1 together with $d_1^\sigma \leq ... \leq d_N^\sigma$ implies that $\theta(t) + d^\sigma$ also satisfies Assumption 1. Hence, the same reasoning as in Section 4.1 establishes that players $l(\Omega_L)$ and $r(\Omega_R)$ are pivotal when the status quo is L and R, respectively. Hence, using Notation 1, $\Theta^\sigma = \Theta(\Omega, d)$. Substituting the latter inequality and (11) into (12), we obtain that $d_n^\sigma = H_n(v, \Omega, d^\sigma)$.

Conversely, let d be a fixed point of H, and let σ be the strategy profile in which each player n votes for R if and only if $\theta(t) + d_n \geq 0$. As shown above, the unique stage-undominated best response for n in some period t to σ is to vote for R if and only if $\theta_n(t) + d_n^\sigma \geq 0$, where d_n^σ is given by (11). To prove that σ is an equilibrium, we need to show that $d_n = d_n^\sigma$ for each $n \in N$.

Equation (9) implies that d_n is increasing in n, and thus by the same reasoning as above, $\Theta^\sigma = \Theta(\Omega, d)$. Clearly, $W_n^\sigma(R) - W_n^\sigma(L)$ must still satisfy (12). Substituting $\Theta^\sigma = \Theta(\Omega, d)$ and the definition of d^σ in (11) on both sides of (12), we obtain that $d_n^\sigma = \frac{\delta v}{1-\delta(1-v)}\int_{\Theta(\Omega,d)}(\theta_n + d_n^\sigma)\, dP(\theta)$. A simple contraction argument shows that this equation has a unique solution d_n^σ, and since by construction d_n is also a solution to that equation, necessarily $d_n = d_n^\sigma$. ■

Proof of Proposition 2. By Lemma 1, in any equilibrium σ, each player n votes for R in some period t if an only if $\theta_n(t) + d_n \geq 0$, where d is such that $d = H(v, \Omega, d)$.

If Ω is quasi-dictatorial, then $l(\Omega_L) = r(\Omega_R)$, so using Notation 1, $\Theta(\Omega, d) = \emptyset$ for all $d \in \mathbb{R}^N$, so $d = H(v, \Omega, d) = (0, ..., 0)$.

Suppose now that Ω is inclusive. One can directly see from (9) that $H_{l(\Omega_L)}(v, \Omega, d) \leq 0 \leq H_{r(\Omega_R)}(v, \Omega, d)$, so $d_{l(\Omega_L)} \leq 0 \leq d_{r(\Omega_R)}$. So $T' \doteq \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} + d_{r(\Omega_R)} > 0\right\}$ contains $T'' \doteq \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} > 0\right\}$. Since $l(\Omega_L) < r(\Omega_R)$, Assumption 1 implies that $P(T'') > 0$, and hence $P(T') > 0$. Since $T' \subset \Theta(\Omega, d)$ and $P(T') > 0$, we have

$$d_{r(\Omega_R)} = H_{r(\Omega_R)}(v, \Omega, d) \geq \frac{\delta v}{1-\delta(1-v)}\int_{T'}\left(\theta_{r(\Omega_R)} + d_{r(\Omega_R)}\right) dP(\theta) > 0.$$

An analogous reasoning implies that $d_{l(\Omega_L)} < 0$, which proves property (ii). Isolating d_n from

$d_n = H_n(v, \Omega, d)$, we obtain

$$d_n = \frac{\int_{\Theta(\Omega,d)} \theta_n dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d))}.$$

From what precedes, $P(\Theta(\Omega, d)) > P(T') > 0$, so Assumption 1 implies that the integral on the numerator of the above equation is strictly increasing in n, which proves property (i). ■

Lemma 2 *Let $(\leq, \geq)_\Omega$ be the partial order on $\mathbb{R}^N$ defined as follows: for all $d, d' \in \mathbb{R}^N$, $d'(\leq, \geq) d$ if $d'_{l(\Omega_L)} \leq d_{l(\Omega_L)}$ and $d'_{r(\Omega_R)} \geq d_{r(\Omega_R)}$. Then the mapping $d \to H(v, \Omega, d)$ is monotonic in the order $(\leq, \geq)_\Omega$.*

Proof. Fix Ω, and since its fixed, let us omit it as the argument in Θ, H, and l and r in this proof. Let $d, d' \in \mathbb{R}^N$ be such that $d'_l \leq d_l$ and $d_r \leq d'_r$. Then using Notation 1, $\Theta(d) \subseteq \Theta(d')$, so

$$H_l(d'_l, d'_r) - H_l(d_l, d_r) = \frac{\delta v}{1 - \delta(1-v)} \left(\int_{\Theta(d)} (d'_l - d_l) dP(\theta) + \int_{\Theta(d') \setminus \Theta(d)} (\theta_l + d'_l) dP(\theta) \right).$$

The integrand of the two integrals on the right-hand side of the above equation are negative on their domains of integration, so $H_l(d'_l, d'_r) \leq H_l(d_l, d_r)$. A similar proof shows that $H_r(d_l, d_r) \leq H_r(d'_l, d'_r)$. ■

Corollary 2 *There exists an equilibrium d^* of $\Gamma(\Omega)$ such that for any other equilibrium d',*

$$d'_{l(\Omega_L)} \leq d^*_{l(\Omega_L)} \leq 0 \leq d^*_{r(\Omega_R)} \leq d'_{r(\Omega_R)}, \tag{13}$$

and for all $n > m$, $0 \leq d^*_n - d^*_m \leq d'_n - d'_m$.

Proof. Let $\|\theta\| \doteq \max_{n \in \mathcal{N}} \int |\theta_n| dP(\theta)$, and for all $d \in \mathbb{R}^N$, let $\|d\| \doteq \max_{n \in \mathcal{N}} |d_n|$. One can see from (9) that for all $d \in \mathbb{R}^N$, $\|H(d)\| \leq \frac{\delta v}{1-\delta(1-v)} (\|\theta\| + \|d\|)$. So if $d = H(d)$, then $\|d\| \leq \frac{\delta v}{1-\delta(1-v)} (\|\theta\| + \|d\|)$, and so $\|d\| \leq \frac{\delta v}{1-\delta} \|\theta\|$. Therefore, all the fixed points of $d \to H(d)$ are in $D \doteq \left\{ d \in \mathbb{R}^N : \|d\| \leq \frac{\delta v}{1-\delta} \|\theta\| \right\}$ and $H(D) \subseteq D$. Since D is a complete lattice for the order $(\leq, \geq)_\Omega$ defined in Lemma 2, Tarski's theorem implies then that the set of fixed points of $d \to H(d)$—and thus the set of equilibria of $\Gamma(\Omega)$—is a complete lattice. As such, it has a minimal element d^* for the order $(\leq, \geq)_\Omega$, which by definition of $(\leq, \geq)_\Omega$ satisfies (13) for any other fixed point d'.

Using Notation 1, (13) implies that $\Theta(\Omega, d^*) \subseteq \Theta(\Omega, d')$. Since $d^* = H(d^*)$ and $d' = H(d')$, the latter inclusion implies that for all $n > m$,

$$\begin{aligned} & ((d'_n - d'_m) - (d^*_n - d^*_m)) \frac{1-\delta(1-v)}{\delta v} \\ = & \int_{\Theta(\Omega,d^*)} (d'_n - d'_m) - (d^*_n - d^*_m)\, dP(\theta) + \int_{\Theta(\Omega,d')\setminus\Theta(\Omega,d^*)} (\theta_n + d'_n - (\theta_m + d'_m))\, dP(\theta), \end{aligned}$$

which implies that

$$\begin{aligned} & (d'_n - d'_m) - (d^*_n - d^*_m) \\ = & \frac{1}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d^*))} \int_{\Theta(\Omega,d')\setminus\Theta(\Omega,d^*)} (\theta_n + d'_n - (\theta_m + d'_m))\, dP(\theta). \end{aligned} \tag{14}$$

By Proposition 2, the integral in (14) is nonnegative, so $d'_n - d'_m \geq d^*_n - d^*_m$. ■

Proof of Proposition 4. When Ω is quasi-dictatorial, from Proposition 2, the four inner inequalities in (8) hold with equality. When Ω is inclusive, Proposition 2 part (i) and (ii) imply that the four inner inequalities in (8) hold strictly.

Before proving the two outer inequalities in (8), let us first prove that

$$d_{l(\Omega'_L)}(\Omega') \leq d_{l(\Omega_L)}(\Omega) \text{ and } d_{r(\Omega_R)}(\Omega) \leq d_{r(\Omega'_R)}(\Omega'). \tag{15}$$

Let $\hat{d} \in \mathbb{R}^N$ be such that $d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)}$ and $\hat{d}_{r(\Omega_R)} \leq d_{r(\Omega'_R)}(\Omega')$. Together with Assumption 1, these inequalities imply that $\Theta\left(\Omega, \hat{d}\right) \subseteq \Theta(\Omega', d(\Omega'))$. Using successively Lemma 1, the latter inclusion, $d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)}$, and Assumption 1, we have

$$\begin{aligned} d_{l(\Omega'_L)}(\Omega') &= \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega',d(\Omega'))} \left(\theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}\right) dP(\theta) \\ &\leq \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega,\hat{d})} \left(\theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}\right) dP(\theta) \\ &\leq \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega,\hat{d})} \left(\theta_{l(\Omega'_L)} + \hat{d}_{l(\Omega'_L)}\right) dP(\theta) = H_{l(\Omega'_L)}\left(v, \Omega, \hat{d}\right). \end{aligned}$$

An analogous argument shows that $d_{r(\Omega'_R)}(\Omega') \geq H_{r(\Omega'_R)}\left(v, \Omega, \hat{d}\right)$. Thus, if we denote $\hat{D} = \left\{\hat{d} \in \mathbb{R}^N : d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)} \text{ and } \hat{d}_{r(\Omega_R)} \leq d_{r(\Omega'_R)}(\Omega')\right\}$, the two previous inequalities imply that $H\left(v, \Omega, \hat{D}\right) \subseteq \hat{D}$. Since the mapping $\hat{d} \to H\left(v, \Omega, \hat{d}\right)$ is monotonic in the order $(\leq, \geq)_\Omega$ defined in Lemma 2, and since $\hat{D}$ is a complete lattice for that order, Tarski's theorem implies

that $\hat{d} \to H\left(v, \Omega, \hat{d}\right)$ admits a fixed point in $\hat{D}$. Clearly, its smallest fixed point, which is $\left(d_{l(\Omega_L)}, d_{r(\Omega_R)}\right)$ by definition, must also be in be in $\hat{D}$. This proves (15).

We are now ready to prove the two outer inequalities in (8). We only prove $d_{r\left(\Omega'_R\right)}(\Omega) \leq d_{r\left(\Omega'_R\right)}(\Omega')$, the argument for $d_{l\left(\Omega'_L\right)}(\Omega') \leq d_{l\left(\Omega'_L\right)}(\Omega)$ is analogous. Assumption 1 and (15) imply that $\Theta(\Omega, d(\Omega)) \subseteq \Theta(\Omega', d(\Omega'))$. Using successively Lemma 1 and the latter inclusion, we get

$$\begin{aligned}
& \left(d_{r\left(\Omega'_R\right)}(\Omega') - d_{r\left(\Omega'_R\right)}(\Omega)\right) \frac{1-\delta(1-v)}{\delta v} \\
= & \int_{\Theta(\Omega', d(\Omega'))} \left(\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega')\right) dP(\theta) - \int_{\Theta(\Omega, d(\Omega))} \left(\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega)\right) dP(\theta) \\
= & \int_{\Theta(\Omega, d(\Omega))} \left(d_{r\left(\Omega'_R\right)}(\Omega') - d_{r\left(\Omega'_R\right)}(\Omega)\right) dP(\theta) + \int_{\Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))} \left(\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega')\right) dP(\theta).
\end{aligned}$$

Isolating $d_{r\left(\Omega'_R\right)}(\Omega') - d_{r\left(\Omega'_R\right)}(\Omega)$, we obtain

$$d_{r\left(\Omega'_R\right)}(\Omega') - d_{r\left(\Omega'_R\right)}(\Omega) = \frac{\int_{\Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))} \left(\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega')\right) dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d(\Omega)))}. \tag{16}$$

By definition of $\Theta(\Omega', d(\Omega'))$, $\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega') \geq 0$ for all $\theta \in \Theta(\Omega', d(\Omega'))$, so the integral in (16) is nonnegative, and thus $d_{r\left(\Omega'_R\right)}(\Omega') \geq d_{r\left(\Omega'_R\right)}(\Omega)$. To complete the proof, it remains to show that when $(l(\Omega_L), r(\Omega_R)) \neq (l(\Omega'_L), r(\Omega'_R))$, this inequality holds strictly, or equivalently, that the integral in (16) is strictly positive.

From (7), $(l(\Omega_L), r(\Omega_R)) \neq (l(\Omega'_L), r(\Omega'_R))$ implies that $r(\Omega_R) < r(\Omega'_R)$ or $l(\Omega_L) > l(\Omega'_L)$. Suppose $r(\Omega_R) < r(\Omega'_R)$, the proof in the other case is analogous. Let

$$T \doteq \left\{\theta \in \mathbb{R}^n : \theta_{l\left(\Omega'_L\right)} + d_{l\left(\Omega'_L\right)}(\Omega') < 0 \text{ and } \theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega') > 0 \text{ and } \theta_{r\left(\Omega'_R\right)} + d_{r(\Omega_R)}(\Omega) < 0\right\}. \tag{17}$$

The first two inequalities in (17) imply that $T \subseteq \Theta(\Omega', d(\Omega'))$. From Assumption 1, the last inequality in (17) implies that $\theta_{r(\Omega_R)} + d_{r(\Omega_R)}(\Omega) < 0$, and thus that T has no intersection with $\Theta(\Omega, d(\Omega))$. Therefore, $T \subseteq \Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))$. Since $\theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega') > 0$ for all $\theta \in T$, the integral in (16) is positive if $P(T) > 0$. From Proposition 2, $d_{l\left(\Omega'_L\right)}(\Omega') \leq 0 \leq d_{r\left(\Omega'_R\right)}(\Omega)$, so since $l(\Omega'_L) < r(\Omega'_R)$, Assumption 1 implies that with probability 1, $\theta_{l\left(\Omega'_L\right)} + d_{l\left(\Omega'_L\right)}(\Omega') < \theta_{r\left(\Omega'_R\right)} + d_{r\left(\Omega'_R\right)}(\Omega)$. Therefore, the first inequality in (17) is redundant with the last, so $T = \left\{\theta \in \mathbb{R}^n : -d_{r\left(\Omega'_R\right)}(\Omega') < \theta_{r\left(\Omega'_R\right)} < -d_{r(\Omega_R)}(\Omega)\right\}$. As shown before, $d_{r\left(\Omega'_R\right)}(\Omega') \geq d_{r\left(\Omega'_R\right)}(\Omega) > d_{r(\Omega_R)}(\Omega)$, so under our assumption that $\theta_{r\left(\Omega'_R\right)}(t)$ has full support, $P(T) > 0$, as needed. ■

Proof of Proposition 3. Using the same steps as for the derivation of (16), we obtain that for all $n > m$,

$$(d_n(\Omega) - d_m(\Omega)) - (d_n(\Omega') - d_m(\Omega')) = \frac{\int_{\Theta(\Omega',d(\Omega'))\setminus\Theta(\Omega,d(\Omega))} (\theta_n + d_n(\Omega') - (\theta_m + d_m(\Omega'))) \, dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d(\Omega)))}.$$

From Assumption 1 and Proposition 2, the integrand of the above integral is positive, so $d_n(\Omega) - d_m(\Omega) \leq (d_n(\Omega') - d_m(\Omega'))$ as needed. Strict inequality follows if $P(\Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))) > 0$, which we have already established in the proof of Proposition 4. ■

Proof of Proposition 6. Fix Ω, and since its fixed, let us omit it as the argument in Θ, H, and l and r. Using the notations of the proposition, let $\hat{d} \in \mathbb{R}^2$ be such that

$$d'_l \leq \hat{d}_l \leq 0 \text{ and } 0 \leq \hat{d}_r \leq d'_r. \tag{18}$$

Applying H to (18) and using Lemma 2 we obtain

$$\begin{cases} H_l(v, d'_l, d'_r) \leq H_l\left(v, \hat{d}_l, \hat{d}_r\right) \leq H_l(v, 0, 0) = 0 \\ H_r(v, d'_l, d'_r) \geq H_r\left(v, \hat{d}_l, \hat{d}_r\right) \geq H_r(v, 0, 0) = 0 \end{cases}.$$

Using successively Lemma 1, $\frac{\partial H_n}{\partial v} = \frac{1-\delta}{(1-\delta(1-v))^2} H_n$, and the above inequalities, we obtain

$$\begin{cases} d'_l = H_l(v', d'_l, d'_r) < H_l(v, d'_l, d'_r) \leq H_l\left(v, \hat{d}_l, \hat{d}_r\right) \leq 0, \\ d'_r = H_r(v', d'_l, d'_r) > H_r(v, d'_l, d'_r) \geq H_r\left(v, \hat{d}_l, \hat{d}_r\right) \geq 0 \end{cases} \tag{19}$$

If we denote $\hat{D} = \left\{\hat{d} \in \mathbb{R}^n : d'_l \leq \hat{d}_l \leq 0 \text{ and } 0 \leq \hat{d}_r \leq d'_r\right\}$, (19) shows that $H\left(v, \hat{D}\right) \subseteq \hat{D}$. Since the mapping $\hat{d} \to H\left(v, \hat{d}\right)$ is monotonic in the order $(\leq, \geq)_\Omega$ defined in Lemma 2, and since $\hat{D}$ is a complete lattice for that order, Tarski's theorem implies that $\hat{d} \to H\left(v, \hat{d}\right)$ admits a fixed point in $\hat{D}$. Clearly, its smallest fixed point, which is (d_l, d_r) by definition, must also be in $\hat{D}$. This means that $d'_l \leq d_l$ and $d_r \leq d'_r$. To show that these inequalities are strict, we need to show that (d'_l, d'_r) cannot be such that $(d'_l, d'_r) = (H_l, H_r)(v, d'_l, d'_r)$, which follows from (19).

When $v = 0$, (9) implies $H(v, d) \equiv (0, ..., 0)$, and thus that $d = (0, ..., 0)$. ■

Proof of Proposition 5. From (6), we have

$$\frac{1}{N}\sum_{n\in\mathcal{N}} d_n = \frac{\delta P(G)}{1-\delta P(G)}\left[\frac{1}{N}\sum_{n\in\mathcal{N}} \bar{\theta}_n + E[\varepsilon|G]\right] = \frac{\delta P(G)}{1-\delta P(G)} E[\varepsilon|G],$$

so the sign of the average distortion is the same as the sign of $E[\varepsilon|G]$. Suppose, by contradiction, that $E[\varepsilon|G] = \frac{\int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l} \varepsilon dF(\varepsilon)}{P(G)} > 0$. Then, by the symmetry of F, $-\bar{\theta}_l - d_l - \bar{\theta}_r - d_r > 0$, which combined with $\bar{\theta}_r + \bar{\theta}_l > 0$, requires that $d_r + d_l < 0$. But from (5),

$$d_l + d_r = \frac{\delta \int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l} \left(\bar{\theta}_l + \bar{\theta}_r + 2\varepsilon\right) dF(\varepsilon)}{1 - \delta \int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l} dF(\varepsilon)} > 0,$$

which is a contradiction. ■

Proof of Proposition 7. Let σ be an equilibrium of the extension considered in Section 5.1. Since σ is Markov, in every t, σ maps the payoff relevant variables $(\theta(t), s(t), e(t), q(t))$ into a distribution over votes $\{L, R\}$. By assumption, $\{(s(t), e(t+1)) : t \in \mathbb{N}\}$ and $\{\theta(t) : t \in \mathbb{N}\}$ are both i.i.d. and independent of each other, so the realization of $e(t)$ and $\theta(t)$ affect the continuation game from period $t+1$ onwards only via their impact on $q(t+1)$. So let $V_c^\sigma(s, q)$ denote expected continuation value for candidate c from period $(t+1)$ onwards, conditional on $s(t) = s$ and $q(t+1) = q$, and on continuation play σ. Stage undomination (together with our tie-breaking rule) implies that in t, σ prescribes c to vote for R if and only if

$$\theta_c(t) + \delta V_c^\sigma(s(t), R) \geq \delta V_c^\sigma(s(t), L).$$

The above inequality shows that candidate c votes for R if and only if $\theta_c(t) + d_c^\sigma(s(t)) \geq 0$ where $d_c^\sigma(s(t)) \equiv \delta\left(V_c^\sigma(s(t), R) - V_c^\sigma(s(t), L)\right)$.

As in the basic model, for a given realization of $(\theta(t+1), e(t+1), s(t+1))$, there are three possibilities: either σ prescribes to a winning coalition of elected candidates to vote for R, in which case R is implemented in $t+1$ irrespective of $q(t+1)$, or σ prescribes to a winning coalition of elected candidates to vote for L, in which case L is implemented in $t+1$ irrespective of $q(t+1)$, or the status quo $q(t+1)$ stays in place in $t+1$. Let Σ^σ denote the set of realizations of $(\theta(t+1), e(t+1), s(t+1))$ that correspond to the last case. Note that $q(t+1)$ affects the policy outcome in $t+1$ only when $(\theta(t+1), e(t+1), s(t+1)) \in \Sigma^\sigma$, and for all such realizations, the status quo stays in place, so the continuation payoff gain of having status quo R instead of L is then $\theta_c(t+1) + \delta\left(V_c^\sigma(s(t+1), R) - V_c^\sigma(s(t+1), L)\right)$. Therefore, if $Q(.|.)$ denotes the probability distribution of $(e(t+1), s(t+1))$ conditional on $s(t)$, we have that for all s^o,

$$V_c^\sigma(s^o, R) - V_c^\sigma(s^o, L) = \int_{\theta, e, s \in \Sigma^\sigma} \left[\theta_c + \delta\left(V_c^\sigma(s, R) - V_c^\sigma(s, L)\right)\right] dP(\theta)\, dQ(e, s|s^o). \quad (20)$$

Let $c, c' \in \mathcal{C}$ be such that $c < c'$. From Assumption 1, $\theta_c(t) < \theta_{c'}(t)$ with probability 1. Together with the equation above, this implies that for all s, $V_c^\sigma(s, R) - V_c^\sigma(s, L) \leq$

$V_{c'}^{\sigma}(s,R) - V_{c'}^{\sigma}(s,L)$ (see footnote 22 in Dziuda and Loeper 2016). Therefore,.as in the basic model, for almost all (s,θ), $\theta_c + d_c^{\sigma}(s) < \theta_{c'} + d_{c'}^{\sigma}(s)$, that is, more rightist candidates vote in a more rightist way. As a result, in every period t, the policy outcome coincides with the vote of the right and left pivots when the status quo is L and R, respectively, where the pivots are defined as in Section 4.1, the only difference being that these pivots now depend on the current profile of elected candidates $e(t)$. If $l(e)$ and $r(e)$ denote these two pivots for a given realization $e \in \mathcal{C}^{\mathcal{N}}$—since Ω is fixed in this section, we omit it from the notations—then

$$\Sigma^{\sigma} = \left\{\theta, e, s : \theta_{l(e)} + d_{l(e)}^{\sigma}(s) < 0 \leq \theta_{r(e)} + d_{r(e)}^{\sigma}(s)\right\}.$$

Therefore, we can rewrite (20) as follows:

$$d_c^{\sigma}(s^o) = \int \left[\int_{\left\{\theta : \theta_{l(e)} + d_{l(e)}^{\sigma}(s) < 0 \leq \theta_{r(e)} + d_{r(e)}^{\sigma}(s)\right\}} \left[\theta_c + \delta d_c^{\sigma}(s)\right] dP(\theta) \right] dQ(e, s|s^o). \tag{21}$$

■

Proof of Proposition 8. Let σ be an equilibrium. When $e(t) \notin D$, the policy that gives the greatest continuation payoff to the common type of the two pivots is implemented irrespective of the status quo. Therefore, one can simplify (20) as follows:

$$d_c^{\sigma}(s^o) = \Pr(e \in D|s^o) \int \left[\int_{\theta_l + d_l^{\sigma}(s) < 0 \leq \theta_r + d_r^{\sigma}(s)} \left[\theta_c + \delta d_c^{\sigma}(s)\right] dP(\theta) \right] dQ(s) = \pi(s^o) d_c^{\sigma}, \tag{22}$$

where the last inequality comes from the fact that the integral on the RHS of (22) is independent of s^o. Note also that the integral inside the bracket in (22) is weakly positive (negative) from $c = r$ $(c = l)$, so $d_l^{\sigma} \leq 0 \leq d_r^{\sigma}$.

Suppose that $\theta_l(t) < 0 < \theta_r(t)$ with positive probability. From what precedes, almost surely,

$$\theta_l + \delta d_l^{\sigma}(s) \leq \theta_l \leq \theta_r \leq \theta_r + \delta d_r^{\sigma}(s).$$

so

$$\int_{\theta_l + d_l^{\sigma}(s) < 0 \leq \theta_r + d_r^{\sigma}(s)} \left[\theta_l + \delta d_l^{\sigma}(s)\right] dP(\theta) \leq \int_{\theta_l < 0 \leq \theta_r} \theta_l dP(\theta) < 0 < \int_{\theta_l < 0 \leq \theta_r} \theta_r dP(\theta) \leq \int_{\theta_l + d_l^{\sigma}(s) < 0 \leq \theta_r + d_r^{\sigma}(s)} [\theta_l$$

which implies $d_l < 0 < d_r$. ■

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