Susceptibility to strategic voting: a comparison of plurality and instant-runoff elections[1]

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Abstract

Advocates of the instant-runoff voting system (IRV, a.k.a. ranked-choice voting) often argue that it is less susceptible to strategic voting than plurality. Is this true? More generally, how should we define and measure a voting system's susceptibility to strategic voting? Previous research in this area is unsatisfying, largely because it ignores the uncertainty voters face when they vote. We introduce a better approach and use it to show that IRV elections are far less susceptible to strategic voting than plurality elections given preference data from 160 election surveys and realistic assumptions about uncertainty and voter behavior. The difference arises partly because, as other voters vote more strategically, strategic voting incentives tend to increase in plurality but decrease in IRV. The methods we introduce can be used to study other properties of voting systems when voters are strategic.

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In recent years, the instant-runoff voting system (IRV) has been a popular reform proposal in several countries. In IRV (also known by several other terms, including ranked-choice voting and the alternative vote[2]), voters rank the candidates and the winner is determined by successively eliminating less-popular candidates. Large-scale referendums to replace plurality voting with IRV failed in the UK in 2011 and British Columbia in 2018 but succeeded in San Francisco in 2002, Maine in 2016, and New York City in 2019. According to its advocates, IRV chooses winners with broader support, discourages negative campaigning, and minimizes strategic voting, among other benefits.[3]

Although many aspects of IRV elections have now been studied (e.g. Farrell and McAllister, 2006; Fraenkel and Grofman, 2006; Horowitz, 2006; McDaniel, 2016; John, Smith and Zack, 2018; McDaniel, 2018), previous research leaves some important questions unanswered. Consider IRV's resistance to strategic voting, which is sometimes offered as its most important advantage over plurality.[4] When a candidate is eliminated in IRV, ballots ranking that candidate first are effectively transferred to the next-ranked candidate. This suggests that the incentive to abandon hopeless candidates is lower in IRV than in plurality. But IRV allows for other types of strategic voting, some rather perverse: for example, one might help elect one's preferred candidate by ranking one's *least*-preferred candidate first (e.g. Fishburn and Brams, 1983; Dummett, 1984). It remains an open question whether, for realistic assumptions about the circumstances voters face in real elections, the opportunities for strategic voting are (as advocates claim) more constrained in IRV than in plurality.

In this paper, we seek to rigorously assess the common claim that IRV is less susceptible to strategic voting than plurality. We define a voting system's susceptibility to strategic voting as the expected benefit to the voter from voting strategically (i.e. casting the expected-utility maximizing vote) instead of voting sincerely (i.e. simply reporting the sincere preference). To

[2] IRV's other names include ranked-choice voting (RCV), the alternative vote (AV), preferential voting, single-transferable vote (STV), and the Hare system. We use instant-runoff voting because the term is widely used and more descriptive than "ranked-choice voting" or "the alternative vote".

[3] See e.g. FairVote's "Ranked Choice Voting 101" at `https://www.fairvote.org/rcv#rcvbenefits`, visited 1 August 2020.

[4] For example, the first argument offered in favor of IRV by UK Deputy Prime Minister Nick Clegg in parliamentary testimony in advance of the 2011 UK referendum was that IRV "stops people from voting tactically and second-guessing how everybody else will vote in their area." See "The Coalition Government's programme of political and constitutional reform: Oral and written evidence", 15 July 2010, HC 358-i, published 22 October 2010 (link).

measure susceptibility to strategic voting, we start with preference data for around 220,000 voters drawn from 160 recent election surveys; we then infer reasonable beliefs these voters might hold about election outcomes in each voting system given others' preferences and a novel model of belief formation informed by empirical measures of forecasting uncertainty. This allows us to measure the expected utility of each possible ballot for each voter under a range of assumptions about the prevalence of strategic voting in the electorate; from this, we extract our measure of each system's susceptibility to strategic voting.

Our analysis shows that, for a wide range of assumptions about how beliefs are formed, IRV is indeed less susceptible to strategic voting than plurality. If we assume that voters have relatively precise beliefs about election outcomes and expect other voters to vote sincerely, the average gain in expected utility from voting strategically (rather than always sincerely) is about five times higher in plurality compared to IRV. The gap becomes wider when we assume that voters have less precise beliefs about election outcomes and/or expect other voters to vote strategically. We find that IRV is less susceptible to strategic voting both because fewer voters benefit from strategic voting (i.e. a sincere vote is optimal for more voters) and because those who expect to benefit from strategic voting benefit by a smaller amount.

Although we are not the first to compare opportunities for strategic voting across voting systems (e.g. Chamberlin and Featherston, 1986; Saari, 1990; Bartholdi and Orlin, 1991; Green-Armytage, Tideman and Cosman, 2016), our approach improves on previous efforts in two significant ways.[5] Most importantly, to our knowledge we are the first to assess susceptibility to strategic voting in a way that takes into account the uncertainty that voters face at the time they decide how to vote. Starting with the Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975), previous research evaluating the possibility of strategic manipulation has assumed that voters know other voters' votes.[6] Ignoring uncertainty simplifies many aspects of the problem, but it may produce misleading conclusions about opportunities for strategic voting in actual elections: even if a voting system sometimes creates situations where a voter

[5] Another contribution is that we use a larger and more representative set of preference distributions, as explained below.

[6] As explained below, this statement applies to research in social choice theory and computational social choice that compares manipulability of voting systems. The study of strategic voting in economics and political science has always taken uncertainty seriously, but it has not sought to measure and compare susceptibility to strategic voting across voting systems.

would *regret* a sincere vote (as Gibbard-Satterthwaite shows it must), it may very rarely create situations where a voter could *anticipate* (given realistic uncertainty) that an insincere vote would be optimal. Indeed, we show that one reason why IRV is less susceptible to strategic voting than plurality is that, given realistic uncertainty, the chance to benefit from an insincere vote in IRV typically comes with a countervailing risk that it backfires.

Our second key innovation is to model voters' beliefs in a way that accounts for strategic behavior by other voters. Previous researchers have assessed opportunities for strategic manipulation assuming that everyone votes sincerely. Again, this simplification may mislead: the incentive to vote strategically may be stronger or weaker when voters take into account others' strategic behavior. To capture a range of possibilities about the prevalence of strategic voting in the electorate, we introduce a model of belief formation in which strategic voters with realistically uncertain beliefs respond myopically to a sequence of polls; we measure the incentive to vote strategically at each stage of this process, with the first iteration capturing the assumption that voters expect others to vote sincerely and later iterations converging on a strategic voting equilibrium.[7] This approach reveals an important qualitative difference between IRV and plurality: in plurality the incentive to vote strategically tends to *increase* as others vote more strategically (because trailing candidates become more hopeless), while in IRV the reverse is true. This tendency toward positive feedback in plurality and negative feedback in IRV is one reason why IRV is less susceptible to strategic voting.

Although this paper focuses on comparing susceptibility to strategic voting in IRV and plurality, we emphasize that the methods we develop are useful for other purposes. Our tools for measuring pivot probabilities in three-candidate IRV elections make it possible to answer other questions about IRV (e.g. assessing the *ex ante* probability of monotonicity failure, studied from an *ex post* perspective by Ornstein and Norman (2014)) and can be extended to handle more candidates. Our iterative polling algorithm can be used to locate an equilibrium in any voting system given arbitrary preferences and offers a way to study other properties of voting systems (such as aggregate voter utility or the probability of electing a Condorcet winner)

[7] Our approach thus includes the assumptions typical of previous work (no uncertainty, sincere voting) as a special case but allows us to relax those assumptions to account for uncertainty and strategic voting by other voters.

while relaxing the assumption that voters vote sincerely. Our entire framework for studying susceptibility to strategic voting can be extended to assess other voting systems. By necessity we do not explore these possibilities in this paper, but we hope to lay a foundation for others to do so.

1 Orientation

1.1 Why measure susceptibility to strategic voting?

Voting theorists have argued that we should prefer a voting system that is less susceptible to strategic voting (e.g. Saari, 1990; Tideman, 2018), meaning a system that is less likely to reward voters who manipulate the result by submitting an insincere preference. Two types of manipulability are potentially concerning.

First, one might be concerned about the *ex post* manipulability of a voting system, by which we mean the frequency with which voters might regret a sincere vote after the results are announced. It is natural and unavoidable for voters who favored unsuccessful candidates to feel disappointment with the outcome, but it seems desirable to avoid situations where voters regret having submitted a sincere ballot, as may have been the case for left-leaning Floridians who voted for Ralph Nader in the 2000 U.S. presidential election. Voters who recognize *ex post* a chance to improve the outcome with an insincere ballot may feel dismay about the vote they or others cast; they may also question the legitimacy of a result that could have been reversed by clever manipulation, particularly when that reversal operates in a perverse way, as when some voters could have obtained a better result by not voting (e.g. Fishburn and Brams, 1983) or by submitting an incomplete ranking of candidates (Fishburn and Brams, 1984).

Second, one can also be concerned about the *ex ante* manipulability of a voting system, meaning the frequency with which voters could obtain a better expected election outcome by submitting an insincere vote at the polls. A system that is more *ex ante* manipulable raises several concerns. To the extent that voters respond to the opportunity to manipulate by submitting insincere votes, the submitted ballots may become more disconnected from the sincere preferences of the electorate, making the election difficult to interpret (Satterthwaite, 1973; Riker, 1982; Green-Armytage, Tideman and Cosman, 2016): did the voters who voted for candidate A really prefer that candidate, or did they vote for A for strategic reasons? Voters also evidently derive expressive benefits from voting for candidates they sincerely support (Hamlin and Jennings, 2011; Pons and Tricaud, 2018), so a system that induces widespread insincere voting also deprives many voters of these expressive benefits. Furthermore, there is evidence that some types of voters (e.g. poorer ones) are less able or inclined to vote strategically (Eggers and Vivyan, 2020), which suggests that a system that is more *ex ante* manipulable disadvantages these voters to a greater extent.[8] Finally, a voting system that is more *ex ante* manipulable creates stronger incentives for voters to pay attention to polls, for media outlets to conduct polls, and for parties to spread polling information (and misinformation); all of this effort might be better spent on other activities, such as scrutinizing candidates' track records or policy proposals (though see Dowding and Van Hees, 2008). Interpretability, expressive utility, fairness, and efficiency arguments thus all favor voting systems that are less *ex ante* manipulable.

1.2 The limitations of previous research

Perhaps surprisingly, previous research on susceptibility to strategic voting focuses exclusively on *ex post* manipulability. The Gibbard-Satterthwaite Theorem states that any reasonable voting system is *ex post* manipulable in some circumstance; the manipulability literature (e.g. Chamberlin and Featherston, 1986; Saari, 1990; Favardin and Lepelley, 2006; Ornstein and Norman, 2014; Green-Armytage, Tideman and Cosman, 2016; Tideman, 2018) quite reasonably builds on this result by measuring the proportion of likely election outcomes that are *ex post* manipulable. More precisely, both the manipulability literature and the social choice literature from which it emerges assume perfect information about the election result, which describes the situation after the election takes place but is plainly unrealistic as a description of the environment in which voters typically decide how to vote. The manipulability literature is also unsatisfying because it checks for opportunities to manipulate when everyone votes sincerely, which is unrealistic at least in plurality elections (where strategic voting is known to be prevalent[9]) even when the

[8] It remains to be seen how heterogeneity in strategic behavior varies across more and less *ex ante* manipulable voting systems, but a voting system in which essentially no one submits insincere votes naturally would have smaller heterogeneity in observed strategic behavior.

[9] See e.g. Kawai and Watanabe (2013).

assumptions about underlying preferences are realistic.

The lack of research into the *ex ante* manipulability of voting systems would be unproblematic if *ex ante* and *ex post* approaches always produced the same conclusions, but they do not. For example, consider a plurality election in which candidates A, B, and C receive nearly equal support, with candidate A defeating candidate B by just one vote and C finishing slightly behind B. Clearly *ex post* manipulation is possible: for example, two supporters of candidate C who prefer B over A could elect B by switching their votes from C to B. But given a poll predicting the same near three-way tie, there is little *ex ante* reason for these voters to switch to B: each pair of candidates is approximately equally likely to be tied for first, so switching from C to B is just as likely to backfire (in the event of a B-C tie) or to lead to a wasted vote (in the event of an A-C tie) as it is to pay off (in the event of an A-B tie). More generally, any system that rewards an insincere vote in one set of circumstances but punishes the same vote in another similar set of circumstances will look more manipulable *ex post* than *ex ante*, because the *ex ante* perspective considers both the benefits and risks of an insincere vote.[10]

By contrast, the game theoretic literature in political science and economics treats strategic voting as an instance of choice under uncertainty (e.g. Myerson and Weber, 1993; Fey, 1997; Myerson, 2002; Myatt, 2007; Laslier, 2009; Bouton, 2013; Bouton and Gratton, 2015) and focuses on characterizing the equilibrium of voting games. To our knowledge, no previous research has applied this literature's view of strategic voting (which takes account of uncertainty and considers how voters' incentives depend on other voters' behavior) to the problem of comparing voting systems' susceptibility to strategic voting. That is our objective in this paper.

2 Susceptibility to strategic voting: a new approach

We seek to measure the extent to which different voting systems encourage strategic voting from an *ex ante* perspective: that is, given information voters might have before an election

[10]The same critique applies to research (mainly in computer science) assessing the complexity of manipulation in various voting systems (see Faliszewski and Procaccia, 2010, for a review). Bartholdi and Orlin (1991) showed that the problem of computing the optimal vote in IRV is NP-complete given perfect information and thus may be intractable as the number of voters and candidates increases. Conitzer, Sandholm and Lang (2007)'s shows that manipulation in IRV given perfect information is *not* hard (in the complexity sense) with a fixed number of candidates, but may be hard when we introduce uncertainty.

takes place, to what extent are voters rewarded for casting an insincere vote? In this section we explain our approach, starting with defining strategic voting and susceptibility to strategic voting before moving to issues of operationalization.

2.1 Strategic voting as expected utility maximization

Following the rational choice tradition (e.g. McKelvey, 1972; Cox, 1997), we view strategic voting as an instance of choice under uncertainty. In that framework, voters have preferences over the candidates competing (which can be captured by a von Neuman-Morgenstern utility function) and beliefs about the probability of various election outcomes (which can be captured by a probability density/mass function). *Voting strategically* means choosing the ballot that, given preferences and beliefs, yields the highest expected utility; by contrast, *voting sincerely* means simply choosing the ballot that most closely matches one's preferences.[11]

More formally, suppose an election takes place involving K candidates $c_1, c_2, \ldots, c_K$ in which voters may submit one of B ballots $b_1, b_2, \ldots, b_B$. Voter i's utility if candidate c_j is elected is given by a utility function $u_i(c_j)$. Let $\mathbf{v}_{-i} = (v_1, v_2, \ldots, v_B)$ be a vector of ballot shares cast by voters other than i, and let $w(\mathbf{v}_{-i}, b_i) \in \{c_1, c_2, \ldots, c_K\}$ denote the winning candidate when i submits b_i and others' votes are given by $\mathbf{v}_{-i}$. Finally, let $f(\mathbf{v}_{-i})$ denote beliefs about election outcomes, specified as a probability mass function defined over the sample space of $\mathbf{v}_{-i}$. Then i's expected utility from casting ballot b_i is

$$\overline{u}_i(b_i) \equiv \sum_{\mathbf{v}_{-i}} u_i \left(w \left(\mathbf{v}_{-i}, b_i \right) \right) f(\mathbf{v}_{-i}) \tag{1}$$

and voting strategically means choosing $b_i^* = \arg\max_{b_i} \overline{u}_i(b_i)$.

2.2 Susceptibility to strategic voting

We will say that a voting system is susceptible to strategic voting to the extent that it puts voters in circumstances where, given information available before the election, they expect to do better with an insincere vote than a sincere one. Compared to other definitions based on the

[11]The two approaches might imply the same action, e.g. when one's favorite candidate is one of the frontrunners in a plurality contest.

possibility of *ex post* manipulation, this definition speaks more directly to concerns that voters might submit insincere ballots (producing difficult-to-interpret election results and depriving voters of the pleasure of expressive voting) or exert effort determining whether they should do so.

Let $\delta_i \equiv \overline{u}_i(b_i^*) - \overline{u}_i(b_i^{\text{sincere}})$ denote the gain voter i receives from voting strategically instead of sincerely (where b_i^* and b_i^{sincere} denote a strategic vote and sincere vote, respectively). This gain is zero if $b_i^* = b_i^{\text{sincere}}$ and positive otherwise. Our main estimand is $E[\delta_i]$, where expectations are taken over circumstances we might expect voters to face, i.e. combinations of preferences and beliefs that might be observed in a given system.

2.3 Estimating susceptibility to strategic voting

Given this definition of susceptibility to strategic voting, we might operationalize and estimate it in various ways. Ideally we might run a large randomized control trial in which we randomly assign voting systems to a large number of polities and, after allowing time for voters and candidates to respond to their assigned voting system, use a survey to measure voters' preferences and beliefs and compare across systems. More realistically, we could carry out observational studies comparing strategic voting incentives across systems in actual use, which comes with the challenge that (especially for uncommon systems) it may be difficult to disentangle differences in susceptibility to strategic voting from differences in the polities where different systems are used.[12] Susceptibility to strategic voting (and voters' responses to strategic voting incentives) could also be studied in the lab (e.g. Blais et al., 2016; Hix, Hortala-Vallve and Riambau-Armet, 2017).

Our approach in this paper is to use a large sample of voters from recent election surveys as the basis for characterizing typical preference distributions; we then posit voter beliefs whose precision is consistent with recent empirical studies and whose location is consistent with the assumed preference distribution and a model of strategic voting. The next few paragraphs explain further our assumptions about preferences and beliefs; Appendix A explains how we

[12]Several studies compare strategic voting in PR and plurality systems (e.g. Bargsted and Kedar, 2009; Abramson et al., 2010). Blais (2004) and Dolez and Laurent (2010) study strategic voting in runoff elections and Farrell and McAllister (2006) discusses instances of of apparent *ex post* manipulability in Australian IRV elections. These studies focus on voter behavior rather than voter incentives.

compute pivot probabilities from beliefs and compute the expected utility of each ballot for each voter in our surveys.

2.3.1 Preferences from election surveys

To capture realistic preferences, we use numerical ratings of parties from 160 national election surveys in 56 unique countries, collected through the Comparative Study of Election Systems (CSES) waves 1-4 (1996-2016).[13] In each survey, respondents are asked to rate each of the main parties on a 0 to 10 scale, where 0 means the respondent "strongly dislikes" that party and 10 means the respondent "strongly likes" that party. We retain these numerical ratings for the three largest parties in each survey (based on national vote share) and add a small amount of random noise (so that there is a unique sincere vote for every voter) to form 160 distinct preference distributions, one for each survey. (Green-Armytage (2014) and Eggers and Vivyan (2020) discuss the suitability of party ratings as utility measures.) The average survey has just under 1,400 respondents who rate all 3 parties, for a total of over 220,000 usable respondents across all the surveys. Because the countries in the survey differ widely in population, and some countries have more surveys in the dataset than others, when we combine results across CSES cases we weight by country population and the number of surveys the country contributes to the CSES,[14] thus characterizing incentives for the typical citizen across the countries in the CSES.

2.3.2 Beliefs from an iterative polling algorithm

Given a distribution of preferences drawn from an election survey, what beliefs (i.e. distribution over possible election results) would it be reasonable to assume? As noted above, the standard approach in manipulability research is to assume perfect information and sincere voting, which in this case would imply assuming that voters in a given survey know that the election outcome will reflect the sincere distribution of preferences in that survey. A game theorist would instead

[13] See http://www.cses.org. There are 162 election surveys in these four waves, but we exclude Belarus in 2008 and Lithuania in 1997 because they record preferences on only two parties.

[14] Specifically, we weight voter i in country j by $\frac{w_i N_j}{n_j}$, where w_i is the normalized survey weight assigned to respondent i (with $\sum w_i = 1$ in each poll), N_j is country j's population, and n_j is the number of surveys from country j in the dataset.

focus on Nash equilibria given some uncertainty about voter actions or the distribution of types: for plurality, this implies assuming that voters in a given survey expect almost everyone to vote for just two candidates.

Rather than focusing on beliefs at either the sincere result or an equilibrium result, we conduct our analysis for a range of beliefs between these extremes. Following Fisher and Myatt (2017), we model voters' common beliefs as a Dirichlet distribution, which can be described by two parameters: the location (or expected value) $\overline{\mathbf{v}} = (\overline{v}_1, \overline{v}_2, \ldots, \overline{v}_B)$, with one share for each of B possible ballots, and a scalar precision parameter s. For each set of analysis we choose a value of s from a range informed by recent empirical work on strategic voting in UK parliamentary elections: Fisher and Myatt (2017) find that voters' beliefs are characterized by $s = 10$, while Eggers and Vivyan (2020) finds that forecasters' beliefs are characterized by $s = 85$.[15] For each election survey, we then trace out a sequence of expected results $\overline{\mathbf{v}}_1, \overline{\mathbf{v}}_2, \ldots$ using a novel iterative polling algorithm as follows.[16] The first expected result $\overline{\mathbf{v}}_1$ is the sincere voting result, i.e. the proportion of respondents in the survey who would cast each ballot if voting sincerely. Each subsequent expected result $\overline{\mathbf{v}}_m$ is a weighted average of the previous result ($\overline{\mathbf{v}}_{m-1}$) and voters' best response to beliefs centered at the previous result (given s), with the weight on voters' best response given by a mixing parameter λ. The sequence may converge on a fixed point, which can be considered a strategic voting equilibrium. We compute each system's susceptibility to strategic voting for beliefs centered at each step along the sequence, which allows readers to assess each voting system under a range of assumptions about the prevalence of strategic behavior.[17]

The iterative polling algorithm can be interpreted in various ways. It can been be seen as a model of inattentive voters responding myopically to a series of polls, starting from a poll of sincere preferences.[18] It could also be seen as a model of levels of rationality (Stahl and Wilson,

[15] As Fisher and Myatt (2017) point out, an observer with an uninformative Dirichlet prior over vote shares who observes a random sample of s voting intentions has Dirichlet posterior beliefs with precision s; thus s can be seen as the size of the poll that informs voter beliefs.

[16] Computer scientists have also written on "iterative voting", which (like our iterative polling algorithm) refers to a procedure in which agents first vote sincerely and then make myopic strategic adjustments (see Meir, 2018, for a review). The difference is that our agents adjust their voting intention based on imprecise beliefs centered at previous poll results, whereas their agents know exactly how others have voted and adjust their votes to achieve a (known) better result.

[17] Note that the algorithm implicitly contains an equilibrium refinement: it selects an equilibrium that can be reached by a process of iterative best responses starting at the sincere result.

[18] In that view, each step in the sequence is a new poll, and a proportion λ of voters note the previous poll and

1994), where the kth result captures the beliefs of level-k voters.[19] More simply, it could be seen as a way to locate a strategic voting equilibrium, which is useful in systems (like IRV) where equilibrium has not yet been characterized. We suspect that observed election results tend to be found somewhere along the sequence produced by the algorithm; we leave for another paper the task of testing that conjecture and exploring the properties of the algorithm more fully.

3 Strategic voting in plurality and IRV elections

To this point we have described a general approach to measuring susceptibility to strategic voting. Before applying this approach to plurality and IRV elections, we briefly discuss the qualitative nature of strategic voting in each system.

To begin with, note that rather than considering all possible election outcomes, a strategic voter can focus on *pivot events* (Myerson and Weber, 1993), i.e. situations where a single vote can determine the winner.[20] That is, the ballot that maximizes expression 1 when we sum over all possible outcomes is the same as the ballot that maximizes a version of expression 1 where we sum over only pivot events. Similarly, the gain from strategic voting compared to sincere voting δ_i is the same whether we compute expected utility over all possible outcomes or only over pivot events.[21] To understand strategic voting in plurality and IRV we will therefore focus on pivot events.

In a three-candidate plurality election, the relevant pivot events are the three possible ties for first.[22] A strategic voter whose preference over candidates is $A \succ B \succ C$ votes B if the probability of a B-C tie for first is sufficiently high relative to an A-B or A-C tie for first. For reasonable specifications of beliefs, a candidate expected to finish third or lower is less likely to be involved in a tie for first than a candidate expected to finish first or second (Fisher and

best respond to it, while a proportion $1 - \lambda$ stick with their previous vote intention.

[19]For example, the first result captures the beliefs of strategic voters who know the distribution of preferences and believe other voters are not strategic.

[20]In terms of the notation above, pivot events describe $\{\mathbf{v}_{-i} : w(\mathbf{v}_{-i}, b_i) \neq w(\mathbf{v}_{-i}, b_i')\}$ for some b_i, b_i'.

[21]We describe how to calculate the probability of any pivot event in IRV and plurality in Appendix A.2.

[22]We could further distinguish between the events where A wins one more vote than B, A and B win the same number of votes, and B wins one more vote than A; for simplicity, suppose that alphabetical order is used to determine the winner if two candidates win the same number of votes, and assume that it is equally likely that A and B have the same number of votes and A leads B by one vote. Then we can refer to a single A-B pivot event.

Myatt, 2017); thus strategic voters tend to abandon trailing candidates, producing Duvergerian results.

Now consider a three-candidate IRV election: the candidate who receives the fewest first-place votes is eliminated, and the winner is the remaining candidate who is ranked higher on the majority of all ballots (including those that ranked the eliminated candidate first). In such an election there are twelve relevant pivot events to consider. There are three pairs of candidates who, having not been eliminated in the "first round", could be tied in the final tally, such that a single ballot could determine the outcome; each of these might be called a "second-round" pivot event. (For example, it could be that C receives the fewest first-place votes and is eliminated, and A is ranked higher than B on exactly half of all ballots.) Second-round pivot events never reward insincere votes: if the winner will be candidate A or B, one cannot do better than submit a sincere ordering of those two candidates. Then there are nine "first-round" pivot events in which a pair of candidates ties for second in top rankings, with the identity of the ultimate winner depending on which one is eliminated. (For example, B and C could be tied for second in first-place votes, and A would win if C were eliminated but C would win if B were eliminated.) First-round pivot events do reward insincere votes. At the pivot event just mentioned, for example, a voter with preference order $A \succ B \succ C$ or $A \succ C \succ B$ could elect A instead of C by ranking the candidates BAC;[23] a voter with preference order $B \succ C \succ A$ could elect C instead of A by ranking the candidates CBA. Note, however, that each of these tactics could backfire if another pivot event took place, including a second-round pivot event.

While there are six ways to rank three candidates, a strategic voter in an IRV election optimally chooses among only three possible ballots: for example, a voter with preference order $A \succ B \succ C$ chooses among ABC, BAC, and CAB. To see why, note that there are first-round pivot events that reward ranking one's second or third choice first (examples appear in the previous paragraph), but the ranking of the other two candidates could only matter in a second-round pivot event, which never rewards an insincere vote.[24]

We may shed additional light on strategic voting in IRV by comparing it to strategic voting

[23]These are illustrations of non-monotonicity, a characteristic of all runoff systems. The tactic is sometimes referred to as voting for a "pushover" (Bouton and Gratton, 2015) or a "turkey" (Cox, 1997).

[24]Dummett (1984) makes this point.

in a conventional runoff election, which has been more extensively studied (e.g. Messner and Polborn, 2007; Bouton, 2013; Ornstein and Norman, 2014; Bouton and Gratton, 2015). The two systems are similar in many respects related to strategic voting, including the possibility of helping one candidate win by ranking another candidate higher. There are two main differences. First, in conventional runoff elections a voter could vote for candidate B in the first round and then, after A and B qualify for the second round, switch to A; this might be attractive if, by voting for B, the voter could prevent C (a more dangerous opponent for A) from advancing. In IRV, by contrast, the same voter can help B advance by submitting a BAC ballot, but this effectively commits the voter to supporting B over A in the second round as well (where the tactic might backfire).[25] Second, in conventional runoff elections voters have an incentive to help a leading candidate secure a majority in the first round; this leads to Duvergerian equilibria in Bouton (2013), with only two candidates winning first-round votes. In an IRV election, where it is not possible to lose support from one round to the next, this incentive is irrelevant. If a candidate is one vote short of a first-round majority, then an additional first-place vote could affect the outcome only if supporters of the third-place candidate *never* rank her second; but then she is also one vote short of a second-round majority, so the circumstance is subsumed in a second-round pivot event.[26]

4 Analysis

4.1 Iterative polling algorithm

We begin by describing the results of the iterative polling algorithm, which provides the sequence of expected results that forms the basis of beliefs in our main analysis. We focus on the case with precision $s = 85$ (the level of precision associated with UK election forecasters by Eggers and Vivyan (2020)) and mixing parameter $\lambda = .05$.[27] A key insight is that the algorithm's ballot shares converge towards a fixed point in both plurality and IRV; while this is well-known

[25] This means that voters in the first round of a conventional runoff do not need to consider second-round pivot events, which tend to constrain insincere voting in IRV.

[26] Put differently, one could consider the event where a candidate only wins if she receives an additional first-round vote as a distinct pivot event, but because it requires two unlikely events (one vote short in first round and no second preferences from the eliminated candidate) it can be safely ignored.

[27] We refer to this parameter combination as the 'baseline' case.

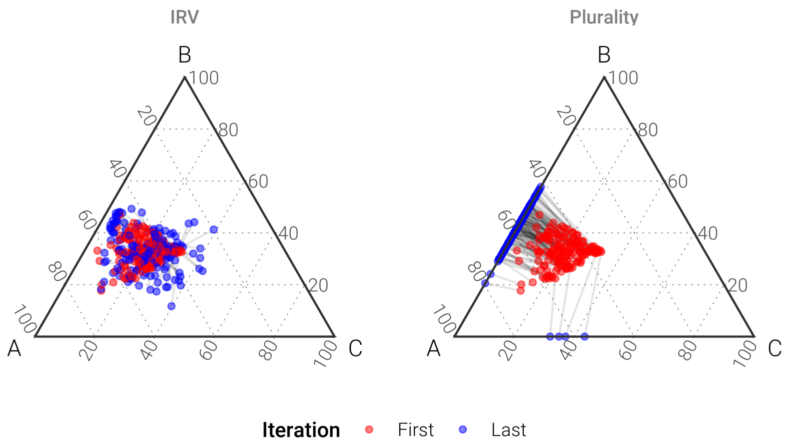


Figure 1: Evolution of ballot share vectors for all 160 CSES election surveys for both IRV (left) and plurality (right). Red dots indicate the first hypothetical poll result, blue dots indicate the 250th hypothetical poll result

and expected in plurality, the algorithm's results in IRV suggest the existence of strategic voting equilibria in IRV, which (to our knowledge) have not yet been documented. Appendix C contains results for other parameter values; we discuss these robustness checks at many points below.

Figure 1 uses a ternary diagram to represent the share of first-preference votes in IRV (left) and plurality (right) in the first hypothetical poll (red dots), i.e. the sincere profile, and the 250th hypothetical poll (blue dots); a gray line traces the intervening polls. In each CSES case we have labeled the parties such that A has the largest share of top rankings and B the second highest; the results of the first poll are therefore all in the lower left corner of the ternary diagram.

In plurality, the iterative polling algorithm traces a path directly from the sincere profile to a Duvergerian equilibrium in which two parties receive all the votes. In almost all cases, the two parties receiving votes are the ones receiving the most sincere preferences (A and B). (The few exceptions were cases where B and C started off nearly tied in sincere preferences and a substantial proportion of voters abandoned B for A, such that B trailed C after a few iterations and subsequently lost all support.) In IRV, by contrast, the iterative polling algorithm in all cases converges on an "interior" point, i.e. one where all candidates receive some first-preference support.

Figure 2 uses a different approach to show that convergence takes place in all CSES cases in both systems. Each line shows, for one CSES election survey, the Euclidean distance between

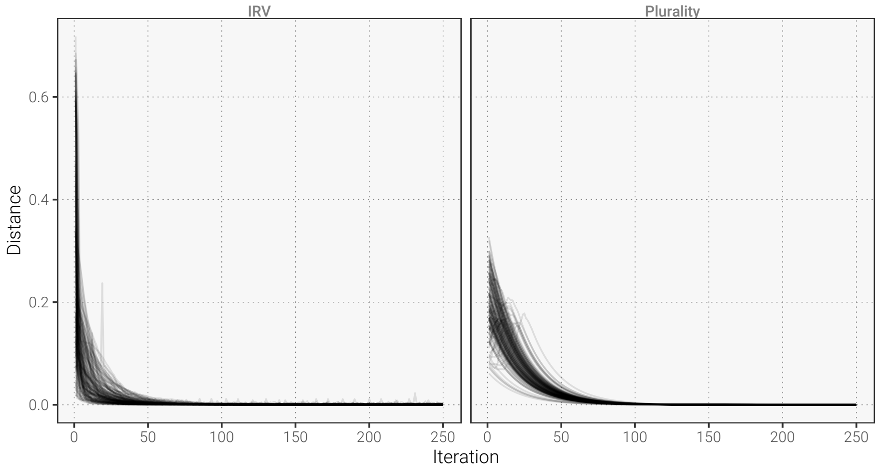


Figure 2: Distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right).

the poll result at a given iteration and the survey respondents' best response to that poll result. (When this distance is zero the algorithm has fully converged.) In both plurality and IRV the distances go to zero in all cases.

Looking closely at Figure 2 we note small oscillations in IRV. Further investigation shows that many IRV cases undergo minor oscillations that highlight the negative feedback we will discuss further below; for example, a poll respondent or group of poll respondents choose to desert a leading candidate in poll m, but this desertion decreases the candidate's lead and causes the same respondent(s) to return to the candidate in poll $m + 1$.[28] As Figure A.15 in the Appendix shows, increasing λ from 0.05 to 0.1 increases the magnitude of these oscillations and (as in plurality) speeds up convergence but does not appear to change the destination of the algorithm.

In plurality, the precision parameter s affects how quickly the algorithm converges to a Duvergerian equilibrium: higher precision makes a vote for the trailing candidate more obviously ineffective and thus speeds desertion of this candidate in favor of the leaders. (See Figures A.4 and A.5 in the Appendix.) In IRV, by contrast, s noticeably affects the location of the equilibrium (as shown by Figures A.14 and A.15). The underlying reason for this difference is that, within the range of s we consider and for results in the neighborhood of the equilibria we find, the choice of s has a larger impact on relative pivot probabilities in IRV than in

[28]It may be many more polls before the previous desertion is "forgotten", causing these voters to desert again.

plurality. Near plurality equilibria a single pivot probability (the probability of a tie between the two frontrunners) dominates all of the others; the choice of s affects how many hundreds or thousands of times larger a tie for first between the frontrunners is than any other tie, but this could only affect the optimal vote for a voter who is nearly indifferent between the frontrunners. Thus a result where two candidates receive all or essentially all of the votes will be a plurality equilibrium at a wide range of s. Near IRV equilibria, by contrast, several pivot probabilities are relevant, and the choice of s can affect not only the relative magnitudes of the pivot probabilities but also their rank ordering. Thus a vote share vector $\overline{\mathbf{v}}$ that is an IRV equilibrium at one value of s will not be an IRV equilibrium at another value of s.

The multiplicity of equilibria in plurality is well known, and can be illustrated with our algorithm: if we replace the starting profile with a result in which candidates B and C are clearly in the lead, for example, we always end up at the equilibrium where those two candidates win all votes. In IRV, by contrast, we find that (for a given value of s) the algorithm converges toward the same point regardless of the starting point, suggesting a single equilibrium. Figure 3 illustrates this for four CSES cases (Australia in 2013, France in 2012, the UK in 2015, and Germany in 2005). In each plot, each red dot indicates a (randomly chosen) starting point, each gray line traces the path of the algorithm, and each blue dot indicates the endpoint after 250 iterations. In each of these CSES cases all of the paths appear to lead to the same point.[29] The apparent uniqueness of equilibrium in IRV across 160 cases strongly suggests (though of course cannot prove) the uniqueness of equilibria in IRV and deserves more study.

4.2 Susceptibility to strategic voting

Our iterative polling algorithm yields a sequence of expected results $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M$ given precision s and mixing parameter λ for each election survey in the CSES. At each stage of each sequence, we compute δ_i (the expected utility benefit from strategic voting vs. sincere voting) for each voter; averaging this across voters gives us a measure of each systems' susceptibility to strategic voting.

[29] There are discrepancies after 250 iterations, but Figure A.16 in the Appendix shows for all CSES cases that all paths from random starting points are still converging toward the endpoint of the baseline algorithm that starts from the sincere profile.

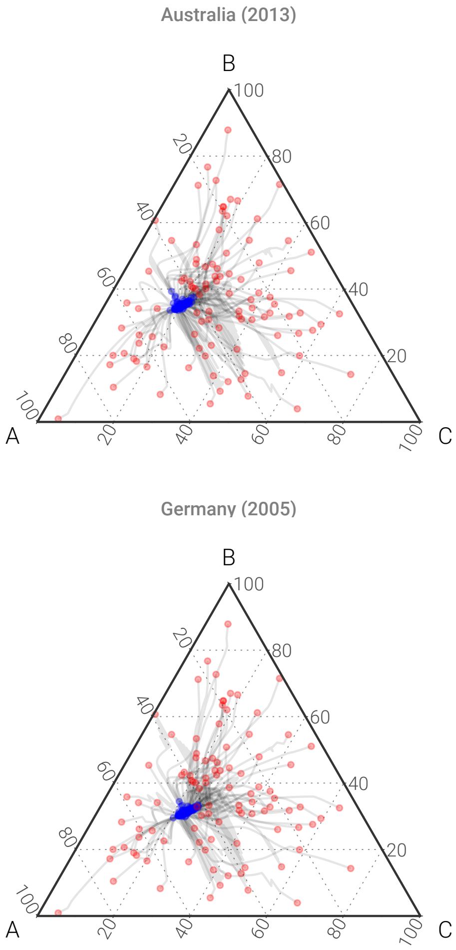


Figure 3: Path of first-preference shares along iterative polling algorithm for select cases in IRV initialised at random starting points in the vote ternary. Red dots indicate starting point, blue dots indicate first-preference shares after 60 iterations.

In Figure 4 we summarize δ_i across our 220,000 CSES respondents in plurality and IRV in three different ways. The plots in the left column focus on $E[\delta_i]$ computed within CSES cases (thin lines) and across all CSES respondents (thick lines, weighted as described in footnote 13) separately for plurality (orange) and IRV (blue) at each of the first 60 iterations[30] of the polling algorithm (horizontal axis) and different values of s ($s = 10$ on top, $s = 55$ in middle, $s = 85$ at

[30]There is less change after the first 60 iterations; the results for all 250 iterations at different combinations of s and λ appear in Appendix D.

bottom). Note that δ_i is measured in the units of the CSES party ratings (where 0 is "strongly dislike" and 10 is "strongly like") multiplied by the assumed size of the electorate; an expected benefit of .4 in an electorate of 1 million, for example, indicates that the average voter would expect to be .4/1,000,000 points (on the 0-10 scale) more pleased with the winner if she were to switch from sincere voting to strategic voting.

The clear conclusion is that IRV is less susceptible to strategic voting than plurality, in the sense that it creates smaller average incentives to vote strategically. At $s = 10$ (approximately the level of belief precision Fisher and Myatt (2017) ascribe to UK voters), the incentive to vote strategically is low for both systems at beliefs close to the sincere profile (i.e. to the left of the diagram), but as voters respond strategically to polls the benefit of strategic voting in plurality increases while the benefit in IRV decreases further. At $s = 85$ the difference in expected benefit is marked even at the sincere profile, grows as voters respond strategically to polls over the first several iterations, and then remains flat with further iterations. More specifically, around the first iteration the expected benefit of strategic voting in plurality is between 5 times higher than in IRV ($s = 85$) and 7 times higher in IRV ($s = 10$); by the 60th iteration the ratio of expected benefits ranges from around 25 (for $s = 85$) to about 35 (for $s = 10$).

Although the degree to which strategic voting incentives are higher in plurality depends on parameters and assumptions, the conclusion that IRV is less susceptible to strategic voting holds across parameters. Notably, across all iterations and choices of s, the expected benefit of strategic voting for plurality in the best case (namely, the first iteration with $s = 10$) is substantially higher than the expected benefit of strategic voting for IRV in the worst case (namely, the first iteration with $s = 85$). Furthermore, sensible departures from our assumptions about belief formation only exacerbate the difference between plurality and IRV: putting some weight on other Duvergerian equilibria in plurality (i.e. those where voters abandon one of the more sincerely popular candidates) would only increase the share of voters who optimally cast an insincere vote, making plurality appear even more susceptible to strategic voting.

To better understand these differences in susceptibility to strategic voting, we decompose our measure into the *magnitude* of the benefit (i.e. how much benefit is there for voters who would benefit from an insincere vote?) and the *prevalence* of the benefit (i.e. what proportion

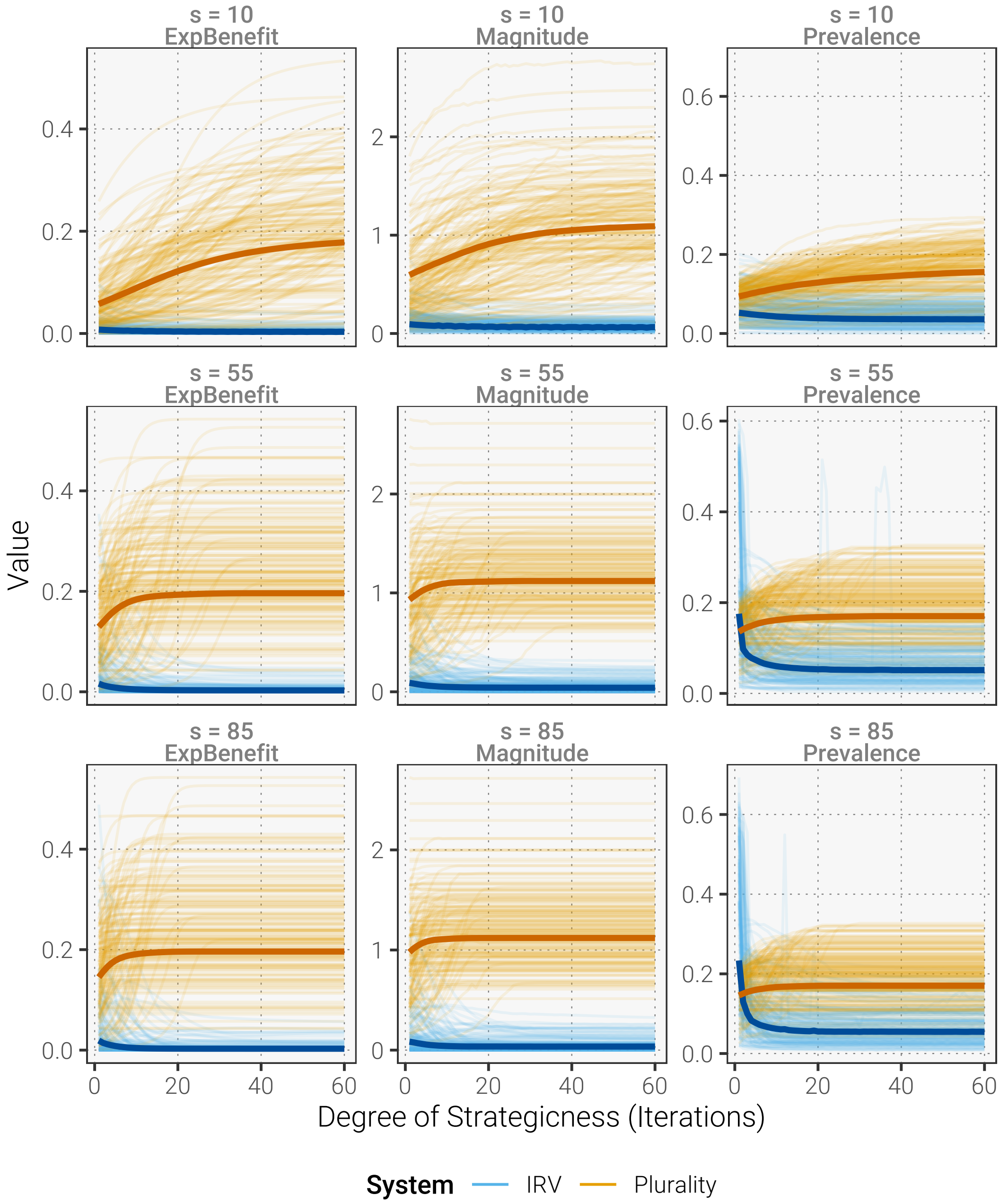


Figure 4: Expected benefit, magnitude, and prevalence of strategic voting

of voters would benefit from an insincere vote?). More formally,

$$\underbrace{E[\delta_i]}_{\substack{\text{Expected}\\ \text{benefit}}} = \underbrace{E[\delta_i \mid \delta_i > 0]}_{\text{Magnitude}} \times \underbrace{E[\mathbb{1}\{\delta_i > 0\}]}_{\text{Prevalence}}.$$

Thus the magnitude corresponds to the intensive margin of δ_i and the prevalence corresponds to the extensive margin of δ_i.

The plots in the second and third columns of Figure 4 show magnitude and prevalence across plurality and IRV for each level of belief precision. The plots indicate that magnitude and prevalence both play a role in producing the overall differences we observe: voters who expect to benefit from an insincere vote do so by less on average in IRV (magnitude) and there are fewer voters who expect to benefit from an insincere vote in IRV (prevalence). Note, however, that in IRV the prevalence near the sincere profile is fairly high: one-fifth or more of voters optimally submit an insincere vote for $s = 55$ and $s = 85$ when they believe other voters will vote sincerely, which is higher than the equilibrium prevalence in plurality. (The magnitude in both cases is much lower in IRV than in plurality.) This points to a difference in the type of feedback that occurs in IRV and plurality: in plurality, the strategic voting incentive is higher on average the more other voters respond to polls; in IRV, it is lower on average. This difference helps explain why IRV is more resistant to strategic voting, as we now explain further.

4.3 Why is plurality more susceptible to strategic voting than IRV?

We emphasize two factors that help explain why plurality is more susceptible to strategic voting than IRV, beginning with the way that strategic voting incentives change with more iterations of the iterative polling algorithm.

Strategic voting shows negative feedback in IRV and positive feedback in plurality

The results in Figure 4 suggest that strategic voting incentives in plurality and IRV depend differently on expectations about others' strategic behavior: strategic voting incentives are highest in IRV when voters expect others to vote sincerely but they are highest in plurality when voters expect others to vote strategically. Why is this the case, and what does it suggest about the likely prevalence of strategic voting in these systems?

The bandwagon logic of strategic voting in plurality is well-understood (e.g. Cox, 1997). If a given candidate is trailing in a poll, then this candidate will trail by even more when other voters respond to the poll; thus if my best naive response to the poll is to abandon a candidate in favor of my second choice, the incentive to do so is likely to be even larger when I take into consideration other voters' responses to the poll.[31] Strategic voting in plurality is thus characterized by positive feedback: strategic responses to a particular pattern of expected results (i.e., one candidate trailing the others) tend to exacerbate that pattern of expected results, inducing more of the same strategic response.[32]

Strategic voting in IRV, by contrast, has a stronger tendency toward negative feedback. To see why, consider a simple example involving three candidates (A, B, and C), with A expected to get the most top ratings; preferences are expected to be mostly single-peaked with B as the centrist, so that B is ranked second on most ballots ranking A or C first. Some of A's supporters may then strategically rank C first, reasoning that C would be an easier second-round opponent (a "turkey" or "pushover") than B would: if B were eliminated her support would be divided between A and C, likely allowing A to maintain her lead, whereas if C were eliminated most of her support would go to B, possibly pushing B ahead of A. But if some A supporters strategically rank C first, A's expected lead over the others narrows, which makes further such desertions less appealing; also, because these desertions increase the share of ballots that rank C first and A second, the ballots become less single-peaked and there is less advantage to A from facing C instead of B in the second round (conditional on a first-round tie between those two candidates). The example highlights the more general tendency toward negative feedback in IRV: strategic responses to a particular pattern of expected results (here, single-peaked preferences with a non-centrist candidate leading) tend to *neutralize* that pattern of expected results, discouraging more of the same strategic response.[33]

[31] If others' desertions widen the expected margin between the top candidates, the effect is ambiguous.

[32] Myatt (2007) emphasizes the role of negative feedback in a model where voters facing a strong coordination incentive receive private signals about which candidate is stronger. Feedback in Myatt (2007) refers not to how voters respond to a public poll but how they respond to their private signal about the popularity of candidates: if other voters respond more to their signal (i.e. vote for the candidate they perceive to be more popular) then it is less important that I do.

[33] Because positive feedback is also possible in IRV (e.g. if B is expected to finish last in the first round and some of C's supporters strategically rank B first to avoid electing A, making it more likely that B and C tie for

This tendency toward negative feedback provides a new perspective on Dummett (1984)'s comment that "a voter who has understood the workings of [IRV], and who has some information about the probable intentions of the others, will have nearly as much incentive to vote strategically" in IRV as in plurality. In light of our analysis, it is true that a strategic voting enthusiast might find ample opportunities for strategic voting in IRV: observing a detailed poll on others' vote intentions, a highly informed voter who thinks herself to be the only strategic actor may often recognize chances to do better in expectation by casting an insincere vote. But the prevalence of negative feedback suggests that this strategic voting incentive typically remains limited to a small proportion of the electorate, because the opportunities to strategically respond to a poll all but disappear when one perceives that others will do so.

Insincere ballots are less likely rewarded in IRV, and benefits and costs more positively associated

Another way to account for the difference in strategic voting incentives we observe is to examine the probability of events that benefit and punish insincere votes. We begin by measuring the average probability of events that reward an insincere vote. The blue line in Figure 5 shows the average probability[34] of ties for first between one's second and third choices in plurality across the first 60 iterations of the algorithm. The probability dips down over the first several iterations as support for C erodes, making it less likely that A and B supporters would benefit from an insincere vote. The red line in Figure 5 shows the average probability of pivot events that reward a ballot ranking one's second choice first in IRV; the green line does the same for pivot events that reward a ballot ranking one's third choice first in IRV.[35] The two IRV lines are considerably below the plurality line, indicating that, averaging across voters, the probability of a pivot event rewarding an insincere ballot is higher on average in plurality than in IRV. This

second), the tendency toward negative feedback is not an intrinsic property of the voting system; it also depends on preferences and beliefs.

[34]For generality, we compute probabilities normalized by electorate size; to get the probability for a given electorate, divide by the electorate size.

[35]For a voter with preference order $A \succ B \succ C$, a ballot ranking B first is better than a sincere ballot if (i) A and B tie for second in the first round and only B would defeat C in the second round, (ii) B and C tie for second in the first round and either would defeat A in the second round, and (iii) B and C tie for second in the first round and only C would defeat A in the second round. A ballot ranking C first is better than a sincere ballot if (i) B and C tie for second in the first round and only B would defeat A in the second round, and (ii) A and B tie for second in the first round and only B would defeat C (so that wasting a ballot by ranking C first is better than voting sincerely).

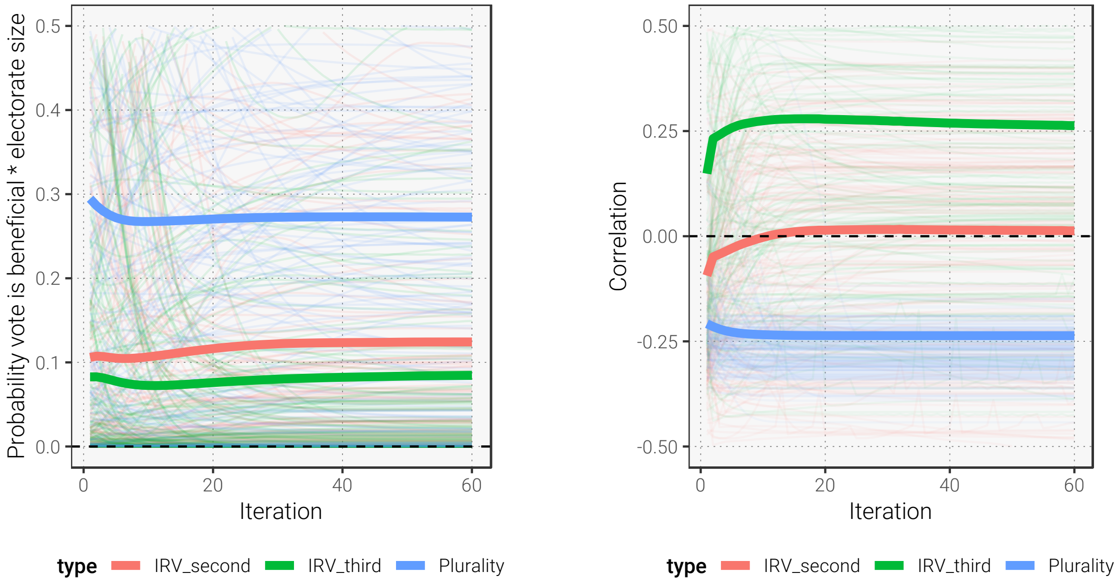


Figure 5: Non-normalized probability that each insincere vote would be rewarded (left) and correlation between probability that each sincere vote would be rewarded and punished (right).

exercise is similar in spirit to manipulability results in e.g. Chamberlin (1985) and Plassmann and Tideman (2014), though we focus on the *ex ante* probability of events that would benefit individual votes (rather than the probability of *ex post* manipulability by groups of voters) and we relax the assumption of sincere voting by other voters.

The right plot in Figure 5 shows that the rewards of insincere voting are also counterbalanced by costs to a greater degree in IRV than in plurality. Having computed the probability of an insincere vote being beneficial for each CSES respondent, we also compute the probability of the same insincere vote being harmful (i.e. worse than a sincere vote[36]) and then computed the correlation in these two probabilities across voters at each iteration. On average, this correlation is negative for plurality (blue line), suggesting that voters who are more likely to see a tie for first between their second and third choice are less likely to see a tie for first involving their first choice. In IRV the average correlation is roughly zero for ranking one's second choice first (red line) but positive for ranking one's third choice first (green line); this suggests that voters who are more likely to be able to elect their first choice by ranking their third choice first are also more likely to see this tactic backfire when e.g. their first choice and third choice tie in the second

[36]For example, voting B in plurality is harmful for a voter with preference order $A \succ B \succ C$ if A ties for first with B or C; voting CAB in IRV is harmful for the same voter at all first-round pivot events where it is not beneficial and at second-round pivot events involving C.

round.[37] The higher correlation between benefits and costs in IRV (shown by Figure 5) suggests that it is not just the proliferation of pivot events (and the resulting complexity of the decision-making process) but also the *density* of pivot events (and the resulting conflicting incentives) that curb strategic voting in IRV. It also highlights the importance of considering uncertainty when assessing voting systems' susceptibility to strategic voting: if we ignore uncertainty and count situations where, *ex post*, a voter could have done better with an insincere vote, we overlook the fact that, from an *ex ante* perspective, this opportunity to do better with an insincere vote may have been counteracted by the real possibility that the vote would backfire. Our analysis shows that this possibility of backfiring is a more important constraint in IRV than in plurality.

5 Conclusion

This paper has introduced a new approach to evaluating voting systems' susceptibility to strategic voting that addresses important shortcomings in previous work. Previous researchers have assessed the manipulability of voting systems by checking how often a voter or group of voters can benefit from an insincere vote assuming perfect information and sincere voting by other voters. We focus instead on measuring strategic voting incentives as they might be perceived by sophisticated voters or elites in advance of an election, which requires allowing for uncertainty and relaxing the assumption that others vote sincerely. Although our method can be used to measure susceptibility to strategic voting in any electoral system, we focus on the contrast between IRV and plurality, as this has been a salient issue in recent electoral reform proposals in the U.S. and elsewhere. In hypothetical three-candidate elections based on preference data from 160 election surveys from 56 countries, we show that plurality is more susceptible to strategic voting than IRV, especially when beliefs are imprecise and strategic voting widespread. We suggest that IRV is less susceptible to strategic voting partly because of negative feedback:

[37] To make sense of this difference between IRV and plurality, consider that an insincere vote is rewarded only when there is relatively even competition across the three candidates: we require a first-round pivot event to take place, which involves a first-round tie for second such that at least one of the trailing candidates can defeat the leader in the second round. But other pivot events that would punish an insincere vote are also more likely when support is fairly balanced across candidates. Insincere votes can backfire in plurality too, of course, but in plurality insincere votes can be rewarded even when such backfiring is a very remote possibility (as when one candidate is far behind the others).

typically, the more I expect others to respond strategically (and myopically) to a poll, the less opportunity for me to benefit from an insincere vote. This contrasts with the well-known bandwagon effect in plurality elections, where the more other voters desert my preferred candidate the stronger is my incentive to do so.

We anticipate that this paper may strike some theoretically-minded readers as too empirical (because it relies on preference data from a specific set of electoral surveys rather than studying voting system properties in a more abstract way) and some empirically-minded readers as too theoretical (because it studies hypothetical elections rather than real ones). To the charge of being too empirical, we respond that (as manipulability research going back to at least Chamberlin (1985) has recognized) one cannot characterize a voting method's susceptibility to strategic voting without specifying preferences; we have used party ratings from electoral surveys to approximate typical preference arrangements, but others are free to apply our methods using other assumptions about preferences. To the charge of being too theoretical, we emphasize the difficulty of empirically comparing the properties of voting systems that are used in disparate situations (or barely used at all); our approach allows us to vary the system while holding fixed important features, though of course we welcome empirical work that responds to and extends our analysis.

We highlight three important caveats to our conclusions. First, susceptibility to strategic voting is only one of many criteria to consider in choosing a voting system; if we focused only on minimizing susceptibility to strategic voting we might end up choosing a voting system that rarely responds to voter preferences (sincere or insincere) at all. Second, we have compared susceptibility to strategic voting while holding fixed voters' preferences over candidates; because the voting system may affect who runs and what positions they adopt, this may give a partial view of the systems' effects on strategic voting incentives as they are experienced by voters. (To the extent that strategic voting in plurality elections discourages good candidates from entering, for example, our approach may overstate the extent to which voters would feel pressured to abandon a preferred candidate.) Bringing these two points together, our measure of susceptibility to strategic voting should be considered along with measures of how the voting system affects candidate entry, candidate positioning, and other outcomes. Third, we have measured susceptibility to strategic voting assuming that voters have imprecise beliefs but otherwise perfectly understand the voting system and can compute the optimal vote given their beliefs; in practice, voters undoubtedly struggle to comprehend complex strategic possibilities and rely heavily on heuristics (Van der Straeten et al., 2010). Our analysis can therefore be seen as measuring hypothetical opportunities for strategic voting, which voters may discover only with the help of elite cues or voting advice applications.

It should be clear that, although we have focused in this paper on susceptibility to strategic voting in three-candidate IRV and plurality elections, the methods we have developed can be applied much more broadly. Not only can we study the same question for other voting systems, more candidates, and/or different preference configurations, but we can also study other properties of voting systems (such as efficiency or the frequency of perverse outcomes) when we relax the usual assumption that voters vote sincerely. In this way, we hope this paper contributes more broadly to the accumulation of knowledge about the tradeoffs involved in the choice of voting systems.

References

Abramson, Paul R, John H Aldrich, André Blais, Matthew Diamond, Abraham Diskin, Indridi H Indridason, Daniel J Lee and Renan Levine. 2010. "Comparing strategic voting under FPTP and PR." *Comparative Political Studies* 43(1):61–90.

Bargsted, Matias A and Orit Kedar. 2009. "Coalition-targeted Duvergerian voting: how expectations affect voter choice under proportional representation." *American Journal of Political Science* 53(2):307–323.

Bartholdi, John J and James B Orlin. 1991. "Single transferable vote resists strategic voting." *Social Choice and Welfare* 8(4):341–354.

Blais, André. 2004. Strategic voting in the 2002 French presidential election. In *The French Voter*. Springer pp. 93–109.

Blais, André, Jean-François Laslier, Karine Van der Straeten et al. 2016. *Voting experiments*. Springer.

Bouton, Laurent. 2013. "A theory of strategic voting in runoff elections." *American Economic Review* 103(4):1248–88.

Bouton, Laurent and Gabriele Gratton. 2015. "Majority runoff elections: strategic voting and Duverger's hypothesis." *Theoretical Economics* 10(2):283–314.

Chamberlin, John R. 1985. "An investigation into the relative manipulability of four voting systems." *Behavioral Science* 30(4):195–203.

Chamberlin, John R and Fran Featherston. 1986. "Selecting a voting system." *The Journal of Politics* 48(2):347–369.

Conitzer, Vincent, Tuomas Sandholm and Jérôme Lang. 2007. "When are elections with few candidates hard to manipulate?" *Journal of the ACM (JACM)* 54(3):14.

Cox, Gary W. 1997. *Making votes count: strategic coordination in the world's electoral systems*. Cambridge University Press.

Dolez, Bernard and Annie Laurent. 2010. "Strategic voting in a semi-presidential system with a two-ballot electoral system. The 2007 French legislative election." *French Politics* 8(1):1–20.

Dowding, Keith and Martin Van Hees. 2008. "In praise of manipulation." *British Journal of Political Science* 38(1):1–15.

Dummett, Michael. 1984. *Voting procedures*. Oxford University Press.

Eggers, Andrew C and Nick Vivyan. 2020. "Who votes more strategically?" *American Political Science Review* . Forthcoming.

Faliszewski, Piotr and Ariel D Procaccia. 2010. "AI's war on manipulation: Are we winning?" *AI Magazine* 31(4):53–64.

Farrell, David M and Ian McAllister. 2006. *The Australian electoral system: origins, variations, and consequences*. UNSW Press.

Favardin, Pierre and Dominique Lepelley. 2006. "Some further results on the manipulability of social choice rules." *Social Choice and Welfare* 26(3):485–509.

Fey, Mark. 1997. "Stability and coordination in Duverger's law: A formal model of pre-election polls and strategic voting." *American Political Science Review* 91(1):135–147.

Fishburn, Peter C and Steven J Brams. 1983. "Paradoxes of preferential voting." *Mathematics Magazine* 56(4):207–214.

Fishburn, Peter C and Steven J Brams. 1984. "Manipulability of voting by sincere truncation of preferences." *Public Choice* 44(3):397–410.

Fisher, Stephen D and D Myatt. 2017. "Strategic voting in plurality rule elections." *Mimeo* .

Fraenkel, Jon and Bernard Grofman. 2006. "Does the alternative vote foster moderation in ethnically divided societies? The case of Fiji." *Comparative Political Studies* 39(5):623–651.

Frigyik, Bela A, Amol Kapila and Maya R Gupta. 2010. Introduction to the Dirichlet distribution and related processes. UWEE Technical Report UWEETR-2010-0006 Department of Electrical Engineering, University of Washington.

Gibbard, Allan. 1973. "Manipulation of voting schemes: a general result." *Econometrica* 41(4):587–601.

Goldberg, David. 1991. "What every computer scientist should know about floating-point arithmetic." *ACM Computing Surveys (CSUR)* 23(1):5–48.

Green-Armytage, James. 2014. "Strategic voting and nomination." *Social Choice and Welfare* 42(1):111–138.

Green-Armytage, James, T Nicolaus Tideman and Rafael Cosman. 2016. "Statistical evaluation of voting rules." *Social Choice and Welfare* 46(1):183–212.

Hamlin, Alan and Colin Jennings. 2011. "Expressive political behaviour: Foundations, scope and implications." *British Journal of Political Science* 41(3):645–670.

Hix, Simon, Rafael Hortala-Vallve and Guillem Riambau-Armet. 2017. "The effects of district magnitude on voting behavior." *The Journal of Politics* 79(1):356–361.

Horowitz, Donald L. 2006. "Strategy takes a holiday: Fraenkel and Grofman on the alternative vote." *Comparative Political Studies* 39(5):652–662.

John, Sarah, Haley Smith and Elizabeth Zack. 2018. "The alternative vote: Do changes in single-member voting systems affect descriptive representation of women and minorities?" *Electoral Studies* 54:90–102.

Kawai, Kei and Yasutora Watanabe. 2013. "Inferring strategic voting." *American Economic Review* 103(2):624–62.

Laslier, Jean-François. 2009. "The leader rule: A model of strategic approval voting in a large electorate." *Journal of Theoretical Politics* 21(1):113–136.

McDaniel, Jason. 2018. "Does More Choice Lead to Reduced Racially Polarized Voting? Assessing the Impact of Ranked-Choice Voting in Mayoral Elections." *California Journal of Politics and Policy* 10(2).

McDaniel, Jason A. 2016. "Writing the rules to rank the candidates: Examining the impact of instant-runoff voting on racial group turnout in San Francisco mayoral elections." *Journal of Urban Affairs* 38(3):387–408.

McKelvey, Richard D. 1972. "A general theory of the calculus of voting." *Mathematical applications in political science* .

Meir, Reshef. 2018. "Strategic Voting." *Synthesis Lectures on Artificial Intelligence and Machine Learning* 13(1):1–167.

Messner, Matthias and Mattias K Polborn. 2007. "Strong and coalition-proof political equilibria under plurality and runoff rule." *International Journal of Game Theory* 35(2):287–314.

Myatt, David P. 2007. "On the theory of strategic voting." *The Review of Economic Studies* 74(1):255–281.

Myerson, Roger B. 2002. "Comparison of scoring rules in Poisson voting games." *Journal of Economic Theory* 103(1):219–251.

Myerson, Roger B and Robert J Weber. 1993. "A theory of voting equilibria." *American Political Science Review* 87(1):102–114.

Ornstein, Joseph T and Robert Z Norman. 2014. "Frequency of monotonicity failure under Instant Runoff Voting: estimates based on a spatial model of elections." *Public Choice* 161(1-2):1–9.

Plassmann, Florenz and T Nicolaus Tideman. 2014. "How frequently do different voting rules encounter voting paradoxes in three-candidate elections?" *Social Choice and Welfare* 42(1):31–75.

Pons, Vincent and Clémence Tricaud. 2018. "Expressive voting and its cost: Evidence from runoffs with two or three candidates." *Econometrica* 86(5):1621–1649.

Riker, William H. 1982. *Liberalism against populism.* San Francisco: WH Freeman.

Saari, Donald G. 1990. "Susceptibility to manipulation." *Public Choice* 64(1):21–41.

Satterthwaite, Mark A. 1973. "The existence of a strategy-Proof voting procedure: A topic in social choice theory." *Ph. D. Dissertation, University of Wisconsin* .

Satterthwaite, Mark Allen. 1975. "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions." *Journal of economic theory* 10(2):187–217.

Stahl, II, Dale O and Paul W Wilson. 1994. "Experimental evidence on players' models of other players." *Journal of economic behavior & organization* 25(3):309–327.

Tideman, Nicolaus. 2018. "How Should Votes Be Cast and Counted?" *The Oxford Handbook of Public Choice* 2:1.

Van der Straeten, Karine, Jean-François Laslier, Nicolas Sauger and André Blais. 2010. "Strategic, sincere, and heuristic voting under four election rules: an experimental study." *Social Choice and Welfare* 35(3):435–472.

Appendix

Table of Contents

A Measuring the expected utility of strategic voting

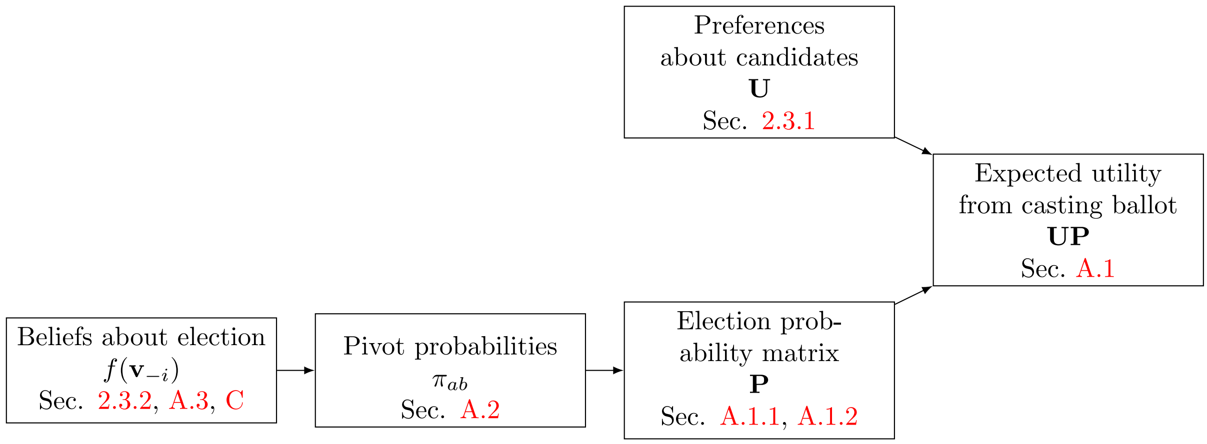


Figure A.1: Steps in calculating voters' expected utility from their preferences and beliefs. The entire process in the figure represents one iteration of our iterative polling algorithm; for the next iteration, beliefs are adjusted accordingly. The cross-references in each box point the reader towards more detail on each step.

In this section, we offer a more technical description of our new method of measuring voters' expected utility from voting strategically. Figure A.1 summarizes the main steps in the process of computing the expected utility of each ballot from preferences and beliefs; each block in the flowchart contains references to sections containing relevant detail for that step.

In what follows, we review the steps in this process roughly in reverse: we show how to compute the expected utility of each ballot in each voting system (assuming known preferences and pivot probabilities); we next show how to compute pivot probabilities given beliefs about likely election outcomes; finally, we describe how we model beliefs about likely election outcomes using in iterative polling algorithm.

A.1 Calculating the expected utility from each possible ballot

Suppose n voters participate in an election to choose a winner from a set of candidates denoted $\mathcal{C}$. We assume that these voters have Von Neumann-Morgenstern utility functions defined over the candidates, with $u_{i,c}$ denoting the utility of voter i from the election of candidate $c \in \mathcal{C}$. We can organize these utilities into a *utility matrix* $\mathbf{U}$ with one row per voter and one column per candidate; for example, given candidates $\{A, B, C\}$, $\mathbf{U}$ is

$$\mathbf{U} = \begin{bmatrix} u_{1A} & u_{1B} & u_{1C} \\ u_{2A} & u_{2B} & u_{2C} \\ \vdots & \vdots & \vdots \\ u_{nA} & u_{nB} & u_{nC} \end{bmatrix}.$$

We also assume that each voter is uncertain about how other voters will vote but all voters share a common belief about the probability of each possible election result, including the election results in which a single ballot could be decisive in various ways, i.e. *pivot events*. Let $\mathcal{B}$ be the set of all permissible ballots (i.e. distinct votes that can be cast) in the voting system. Let $p_{c,b}$ be the probability that candidate c is elected given one additional ballot $b \in \mathcal{B}$ is submitted, and organize these probabilities into an *election probability matrix* $\mathbf{P}$ with one row per candidate and one column per ballot (so its dimensions are $|\mathcal{C}| \times |\mathcal{B}|$). Then the expected utility of each voter from submitting each possible ballot is the *expected utility matrix* $\overline{\mathbf{U}} = \mathbf{UP}$ with n rows (one row per voter) and one column per ballot. From the expected utility matrix we can compute the optimal (strategic) ballot for each of our n voters as well as the difference in each voter's expected utility between casting the optimal ballot and casting a sincere ballot, which is a measure of the voter's strategic voting incentive. Studying strategic voting incentives in any voting system given voters' preferences and beliefs is thus essentially a problem of assembling the election probability matrix $\mathbf{P}$. We now show how to do this in three-candidate plurality and IRV elections given the probability of pivot events (i.e. *pivot probabilities*); later we show how to compute these pivot probabilities given beliefs.

A.1.1 The P matrix in plurality

In plurality, voters submit ballots naming one candidate, so the set of admissible ballots is the set of candidates. Given candidates $\{A, B, C\}$, the $\mathbf{P}$ matrix is

$$\mathbf{P} = \begin{bmatrix} p_{A,A} & p_{A,B} & p_{A,C} \\ p_{B,A} & p_{B,B} & p_{B,C} \\ p_{C,A} & p_{C,B} & p_{C,C} \end{bmatrix},$$

where e.g. $p_{B,A}$ indicates the probability B is elected when one votes for A. Let an election result be written as a vector $\mathbf{v} = (v_A, v_B, v_C)$, with e.g. v_A indicating the share of ballots naming candidate A. We assume throughout that voters consider $\mathbf{v}$ to be a continuous random variable, with beliefs summarized by pdf $f(\mathbf{v})$; this simplifies the analysis by eliminating the possibility of ties. Given a total electorate of size N,[38] the probability that a single ballot could change the plurality winner from candidate j to candidate i is then

$$\pi_{ij} = \Pr\left(v_j - v_i \in \left(0, N^{-1}\right) \cap v_i > v_k\right). \tag{2}$$

Assuming each candidate is equally likely to finish just ahead of or just behind another candidate (so that $\pi_{ij} = \pi_{ji}$), the diagonal elements of $\mathbf{P}$ in plurality are

$$p_{i,i} = \pi_i + 2(\pi_{ij} + \pi_{ik}), \tag{3}$$

where i, j, and k are distinct candidates. That is, i wins (given an additional vote for i) if i would win in any case (which occurs with probability π_i), if i would finish either slightly behind or slightly ahead of j ($2\pi_{ij}$), or if i would finish either slightly behind or slightly ahead of k ($2\pi_{ik}$). The off-diagonal elements are

$$p_{j,i} = \pi_j + \pi_{jk} \tag{4}$$

where again i, j, and k are distinct candidates. That is, j wins (given an additional vote for i) if j would win in any case (π_j) or if j is slightly ahead of k (π_{jk}).

It will be convenient to work with a normalized version of $\mathbf{P}$ in which we set π_i to 0 for $i \in \{A, B, C\}$, thus ignoring results in which a single ballot could not determine the outcome. In that case $\mathbf{UP}$ produces a normalized (i.e. recentered) measure of expected utility that is sufficient for determining both the optimal ballot and the benefit of strategic voting.[39]

[38] Thus the n voters may be a sample of the larger electorate.

[39] Similarly, Myerson and Weber (1993) focus on the gain in expected utility relative to abstention.

Table 1: Pivot events in IRV

Label	Type	Description	Probability
$ij.2$	Second-round	i and j tie after k is eliminated in 1st round	$\pi_{ij.2} = \Pr\left(v_j + v_{kj} - \frac{1}{2} \in \left(0, N^{-1}\right) \cap v_k < v_i \cap v_k < v_j\right)$
$ij.ik$	First-round	i and j tie for 2nd in 1st round; only i would defeat k	$\pi_{ij.ik} = \Pr\left(v_j - v_i \in \left(0, N^{-1}\right) \cap v_j < v_k \cap v_k + v_{ik} > \frac{1}{2} \cap v_k + v_{jk} < \frac{1}{2}\right)$
$ij.kj$	First-round	i and j tie for 2nd in 1st round; only j would defeat k	$\pi_{ij.kj} = \Pr\left(v_j - v_i \in \left(0, N^{-1}\right) \cap v_j < v_k \cap v_k + v_{ik} < \frac{1}{2} \cap v_k + v_{jk} > \frac{1}{2}\right)$
$ij.ij$	First-round	i and j tie for 2nd in 1st round; both i and j would defeat k	$\pi_{ij.ij} = \Pr\left(v_j - v_i \in \left(0, N^{-1}\right) \cap v_j < v_k \cap v_k + v_{ik} < \frac{1}{2} \cap v_k + v_{jk} < \frac{1}{2}\right)$

Notes: In this table a "tie" indicates that one candidate finishes slightly ahead of the other, such that a single ballot could reverse the order of finish.

A.1.2 The P matrix in IRV

In an IRV election involving three candidates $\{A, B, C\}$, voters submit ballots ranking the candidates, so the admissible ballots are $\{AB, AC, BA, BC, CA, CB\}$ (where ij denotes a ballot that ranks candidate i first, j second, and (implicitly) k third). The **P** matrix then looks like

$$\mathbf{P} = \begin{bmatrix} p_{A,AB} & p_{A,AC} & p_{A,BA} & p_{A,BC} & p_{A,CA} & p_{A,CB} \\ p_{B,AB} & p_{B,AC} & p_{B,BA} & p_{B,BC} & p_{B,CA} & p_{B,CB} \\ p_{C,AB} & p_{C,AC} & p_{C,BA} & p_{C,BC} & p_{C,CA} & p_{C,CB} \end{bmatrix}$$

and an election result can be written as $\mathbf{v} = (v_{AB}, v_{AC}, v_{BA}, v_{BC}, v_{CA}, v_{CB})$.

A three-candidate IRV election can be considered to take place in two rounds: in the first round the candidate who receives the fewest first-place votes is eliminated; in the second round the winner is determined based on the ranking of the remaining two candidates on all ballots.[40] There are two classes of pivot events in IRV. In *second-round pivot events*, a single ballot determines who wins the second round. Let $ij.2$ denote the event that a single ballot ranking i above j could change the IRV winner from j to i in the second round, which (again assuming $\mathbf{v}$ is continuous) occurs when j is preferred to i on only slightly more than half of all ballots and k receives fewer top rankings than either i or j. The probability of this pivot event ($\pi_{ij.2}$) appears in the first row of Table 1. In *first-round pivot events* a single ballot determines the winner by determining who advances to the second round. If candidates i and j are essentially tied for second (in top rankings) in the first round, such that a single ballot determines which one advances, then there are three scenarios in which a single ballot could determine the winner: when either candidate (i or j) would defeat k in the second round (event $ij.ij$), when only i would defeat k in the second round (event $ij.ik$), and when only j would defeat k in the second round (event $ij.kj$). These events are described in Table 1, with the associated probability appearing in the final column.

To fill in the **P** matrix for IRV using pivot probabilities, we assume again that adjacent pivot events are equally likely: $\pi_{ij.2} = \pi_{ji.2}$ (i.e. each candidate is just as likely to trail as to lead another candidate in the second round) and $\pi_{ij.ik} = \pi_{ji.ki}$, $\pi_{ij.kj} = \pi_{ji.kj}$, and $\pi_{ij.ij} = \pi_{ji.ji}$ (i.e. each candidate is just as likely to trail as to lead another candidate for second in the first

[40]Descriptions of three-candidate IRV often note that the election ends in the first round if one candidate wins a majority of top rankings, but such a candidate would obviously win the second round so this step is superfluous.

round, for each possible way that the first round outcome could determine the winner). Then we have

$$p_{i,ij} = p_{i,ik} = \pi_i + 2(\pi_{ij.2} + \pi_{ik.2} + \pi_{ij.ij} + \pi_{ij.ik} + \pi_{ik.ik} + \pi_{ik.ij}) + \pi_{jk.ik} + \pi_{jk.ji},$$

meaning that i wins (given an additional ballot ranking i first) if i would win in any case (which occurs with probability π_i); if i would finish nearly tied with (i.e. just ahead of or just behind) j or k in the second round ($2(\pi_{ij.2} + \pi_{ik.2})$); if i would finish nearly tied with j or k for second in the first round and would win if it advanced ($2(\pi_{ij.ij} + \pi_{ij.ik} + \pi_{ik.ik} + \pi_{ik.ij})$); or if j and k would nearly tie for second in the first round, only one of them would lose to i in the second round, and that candidate is the one who advances ($\pi_{jk.ik} + \pi_{jk.ji}$). Similarly, we have

$$\begin{aligned} p_{i,jk} &= \pi_i + 2\pi_{jk.ik} + \pi_{ik.ik} + \pi_{ik.ij} \\ p_{i,ji} &= \pi_i + 2(\pi_{ik.2} + \pi_{jk.ik}) + \pi_{ik.ik} + \pi_{ik.ij}. \end{aligned}$$

The first expression states that i wins (given an additional ballot ranking j first and k second) if i would win in any case (which occurs with probability π_i); if j and k would nearly tie for second in the first round and only k would defeat i ($2\pi_{jk.ik}$), so that a ballot of ji ensures i's victory; or if i would finish the first round narrowly ahead of k for second place and would defeat j in the second round ($\pi_{ik.ik} + \pi_{ik.ij}$). The second expression states that i wins (given an additional ballot ranking j first and i second) in all the same situations plus when i would finish nearly tied with k in the second round ($2\pi_{ik.2}$). As explained above, in practice we set π_i to zero for $\{A, B, C\}$, which focuses on pivot events and produces a normalized measure of expected utility.

A.2 Computing pivot probabilities

Next, we show how to compute pivot probabilities in three-candidate plurality and IRV elections in order to be able to compute the election probability matrix $\mathbf{P}$.

In both systems, we assume that the distribution over election outcomes $f(\mathbf{v_{-i}})$ follows a Dirichlet distribution, which implies modeling the distribution of votes as a continuous random variable. One benefit of this assumption is that the probability of an exact tie between two candidates is zero, so we can avoid tedious complications about tie-breaking and making vs. breaking ties. The Dirichlet distribution's properties make it particularly convenient to work with, as we will see when we compute pivot probability for IRV elections.

A.2.1 Pivot Probabilities in Plurality

A single vote can move candidate j ahead of candidate i when $v_i - v_j \in (0, \frac{1}{N})$, where N is the total size of the electorate. Without loss of generality, the probability that a single vote can elect candidate 2 instead of 1 in a three-candidate race is

$$\pi_{12} = \Pr\left(v_1 - v_2 \in \left(0, \frac{1}{N}\right) \cap v_1 > v_3\right)$$

which given $f(\mathbf{v_{-i}})$ and an electorate of size N can be written as

$$\pi_{12} = \int_{\frac{1}{3}}^{\frac{1}{2}} \int_{v_1 - \frac{1}{N}}^{v_1} f(v_1, v_2, 1 - v_1 - v_2)\, dv_2\, dv_1.$$

If $f(\mathbf{v_{-i}})$ is smooth and the electorate N is large, then

$$\pi_{12} \approx \frac{1}{N} \int_{\frac{1}{3}}^{\frac{1}{2}} f(y, y, 1 - 2y)\, dy. \tag{5}$$

(Note that the same approximation would hold for the probability that a single vote can elect candidate 1 instead of 2, so we can collapse these events into a single pivot event π_{12}.) This approximation can be computed by numerical integration methods.[41]

A.2.2 Pivot Probabilities in IRV

To compute the probability of pivot events in IRV given Dirichlet beliefs, we make use of three well-known (Frigyik, Kapila and Gupta, 2010) properties of the Dirichlet distribution:

Aggregation property: $(v_1, v_2, \ldots, v_i + v_j, \ldots v_B) \sim \mathrm{Dir}(\alpha_1, \alpha_2, \ldots, \alpha_i + \alpha_j, \ldots \alpha_B)$. (If two of the vote shares are added together to create a new, shorter vector of vote shares, the new vector of vote shares also follows a Dirichlet distribution, where the parameters corresponding to the summed-up vote shares are also summed up.)

Marginal distribution: $v_i \sim \mathrm{Beta}(\alpha_i, \sum_{-i} \alpha)$. (Unconditionally, any particular vote share follows a Beta distribution. This follows from the aggregation property and the observation that a Dirichlet distribution with two parameters is a Beta distribution.)

Conditional distribution: $(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_B \,|\, v_i) \sim (1-v_i)\mathrm{Dir}(\alpha_1, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_B)$. (Conditional on i receiving share v_i, the remaining shares follow a rescaled Dirichlet distribution in which α_i is removed from the parameter vector.)

[41] Fisher and Myatt (2017) provide an analytical expression for relative pivot probabilities in three-candidate plurality contests given Dirichlet beliefs. Eggers and Vivyan (2020) validate a numerical approximation when there are more than three candidates.

We will use $f(\mathbf{v}; s\overline{\mathbf{v}})$ to indicate the Dirichlet density with parameters $s\overline{\mathbf{v}}$ evaluated at $\mathbf{v}$. Because the Beta density can be seen as a special case of the Dirichlet density, we will use $f(\cdot)$ for both. As in the main text, v_{ab} denotes the share of ballots ranking a first, b second, and (implicitly) c third; with v_{ac}, v_{ba} etc similar; v_a denotes the share of ballots listing a first, i.e. $v_a \equiv v_{ab} + v_{ac}$.

Probability of second-round pivot events: The probability of a trailing b by less than $\frac{1}{N}$ in the second round can be written

$$\Pr\left(v_c < v_a < \frac{1}{2} \cap v_c < v_b < \frac{1}{2} \cap v_a + v_{ca} - \frac{1}{2} \in \left(-\frac{1}{N}, 0\right)\right).$$

This can be factorized as

$$\Pr\left(v_a + v_{ca} - \frac{1}{2} \in \left(-\frac{1}{N}, 0\right)\right) \times \Pr\left(v_c < v_a \cap v_c < v_b \,\middle|\, v_a + v_{ca} - \frac{1}{2} \in \left(-\frac{1}{N}, 0\right)\right). \tag{6}$$

Using the aggregation property, the first term in expression 6 is

$$\int_{-\frac{1}{N}}^{0} \int_{0}^{\frac{1}{2}} f\left(y - x/4, \frac{1}{2} - y - x/4, \frac{1}{2} + x/2; s\overline{v}_a, s\overline{v}_{ca}, s(\overline{v}_b + \overline{v}_{cb})\right) dy\, dx$$

which is approximately

$$\frac{\sqrt{6}}{4}\frac{1}{N}\int_{0}^{\frac{1}{2}} f\left(y, \frac{1}{2} - y, \frac{1}{2}; s\overline{v}_a, s\overline{v}_{ca}, s(\overline{v}_b + \overline{v}_{cb})\right) dy.$$

(The approximation is exact if the density is flat in the immediate neighborhood of second-round ties between a and b.) To understand the leading $\frac{\sqrt{6}}{4}$ term: On the 3-dimensional unit simplex with vertices v_a, v_{ca}, and $1 - v_a - v_{ca}$, draw a line where $v_a + v_{ca} = \frac{1}{2}$; this locus characterizes pairwise ties between a and b and goes through the point $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. Now draw a line parallel to the first but shifted slightly so that it runs through the point $(\frac{1}{4} - \frac{1}{4N}, \frac{1}{4} - \frac{1}{4N}, \frac{1}{2} - \frac{1}{2N})$. Between the two lines is the narrow strip where b wins a pairwise contest between the two candidates but a single ballot could move a ahead of b. The width of this strip is $\sqrt{\left(\frac{1}{4N}\right)^2 + \left(\frac{1}{4N}\right)^2 + \left(\frac{1}{2N}\right)^2} = \frac{\sqrt{6}}{4N}$.

We now turn to the second term in expression 6. Given that $v_a = y$, $v_{ca} = \frac{1}{2} - y$, and $v_b + v_{cb} = \frac{1}{2}$, we note that $v_c < v_a$ implies $v_{cb} < 2y - \frac{1}{2}$ and $v_c < v_b$ implies $v_{cb} < \frac{y}{2}$; comparing the two conditions, note that the former binds when $y < \frac{1}{3}$ and the latter binds otherwise. Next, using all three properties of the Dirichlet noted above and given that $v_a + v_{ca} = \frac{1}{2}$,

$$(v_{cb} \mid v_a + v_{ca}) \sim \frac{1}{2}\text{Beta}\left(s\overline{v}_{cb}, s\overline{v}_b\right), \tag{7}$$

i.e. given that half the ballots list a first or list c first and a second, the proportion listing c first and b second (instead of b first) lies between 0 and 1/2; if we multiply the proportion by two, the result is distributed according to a Beta distribution with parameters $s\overline{v}_{cb}$ and $s\overline{v}_b$. Thus to find the probability that $v_{cb} < 2y - \frac{1}{2}$ (the binding constraint in the second term from expression 6 when $y < 1/3$), we integrate this distribution from 0 to $2y - \frac{1}{2}$; to find the probability that $v_{cb} < \frac{y}{2}$ (the binding constraint in the second term from expression 6 when $y > 1/3$), we integrate this distribution from 0 to $\frac{y}{2}$. Finally note that y (i.e. v_a) cannot be below 1/4; otherwise either a finishes last in first-preference votes or b receives more than half of first-preference votes. Combining all of this, we have

$$\begin{aligned} N\pi_{ab} &\approx \frac{\sqrt{6}}{4}\int_{\frac{1}{4}}^{\frac{1}{3}} f\left(y, \frac{1}{2}-y, \frac{1}{2}; s\overline{v}_a, s\overline{v}_{ca}, s(\overline{v}_b+\overline{v}_{cb})\right)\int_0^{2y-\frac{1}{2}} f\left(2z, 1-2z; s\overline{v}_{cb}, s\overline{v}_b\right) dz\, dy + \\ &\int_{\frac{1}{3}}^{\frac{1}{2}} f\left(y, \frac{1}{2}-y, \frac{1}{2}; s\overline{v}_a, s\overline{v}_{ca}, s(\overline{v}_b+\overline{v}_{cb})\right)\int_0^{\frac{y}{2}} f\left(2z, 1-2z; s\overline{v}_{cb}, s\overline{v}_b\right) dz\, dy. \end{aligned} \quad (8)$$

Note that the second and fourth densities are evaluated at $(v_{cb} = 2z, v_b = 1-2z)$ rather than $(v_{cb} = z, v_b = \frac{1}{2} - z)$ because of the $\frac{1}{2}$ in expression 7.

The analysis extends straightforwardly to the two other second-round pivot events by exchanging candidate labels.

Probability of first-round pivot events: First-round pivot event $ab.ab$ takes place when a ties b for second place in first-preference votes and either candidate would win the election if the other were eliminated. Generally, the probability of $ab.ab$ is

$$\Pr\left(v_b - v_a \in \left(-\frac{1}{2N}, \frac{1}{2N}\right) \cap v_b < v_c \cap v_a < v_c < \frac{1}{2} \cap v_a + v_{ba} > v_c + v_{bc} \cap v_b + v_{ab} > v_c + v_{ac}\right), \quad (9)$$

which can be factorized as

$$\begin{aligned} &\Pr\left(v_b - v_a \in \left(-\frac{1}{2n}, \frac{1}{2n}\right) \cap v_b < v_c \cap v_a < v_c < \frac{1}{2}\right) \times \\ &\Pr\left(v_a + v_{ba} > v_c + v_{bc} \cap v_b + v_{ab} > v_c + v_{ac} \,\middle|\, v_b - v_a \in \left(-\frac{1}{2N}, \frac{1}{2N}\right) \cap v_b < v_c \cap v_a < v_c < \frac{1}{2}\right). \end{aligned}$$

Using the same approximation as above, the first line is approximately

$$\frac{1}{\sqrt{2}}\frac{1}{N}\int_{\frac{1}{4}}^{\frac{1}{3}} f\Big(z, z, 1-2z; s\overline{v}_a, s\overline{v}_b, s\overline{v}_c\Big) dz.$$

Letting $v_a = v_b = z \in \left(\frac{1}{4}, \frac{1}{3}\right)$, the second term becomes

$$\Pr\left(v_{bc} < 2z - \frac{1}{2} \cap v_{ac} < 2z - \frac{1}{2} \,\middle|\, v_a = v_b = z\right). \quad (10)$$

and again combining all three properties we have

$$\begin{aligned} (v_{bc}|v_a + v_c) &\sim z\text{Beta}\big(s\overline{v}_{bc}, s\overline{v}_{ba}\big) \\ (v_{ac}|v_b + v_c) &\sim z\text{Beta}\big(s\overline{v}_{ac}, s\overline{v}_{ab}\big). \end{aligned}$$

Putting together the above, we have

$$\begin{aligned} N\pi_{ab.ab} &\approx \frac{1}{\sqrt{2}}\int_{\frac{1}{4}}^{\frac{1}{3}} f\Big(z, z, 1-2z; s\overline{v}_a, s\overline{v}_b, s\overline{v}_c\Big) \times \\ &\int_0^{2z-\frac{1}{2}} f\left(\frac{x}{z}, \frac{z-x}{z}; s\overline{v}_{bc}, s\overline{v}_{ba}\right) dx \times \int_0^{2z-\frac{1}{2}} f\left(\frac{x}{z}, \frac{z-x}{z}; s\overline{v}_{ac}, s\overline{v}_{ab}\right) dx\, dz \end{aligned} \quad (11)$$

To get the probability of pivotal event $ab.ac$ we reverse the last inequality in expression 9 (changing $v_b + v_{ab} > v_c + v_{ac}$ to $v_b + v_{ab} < v_c + v_{ac}$), which means changing the last term in expression 11 from $\int_0^{2z-\frac{1}{2}} f\left(\frac{x}{z}, \frac{z-x}{z}; s\overline{v}_{ac}, s\overline{v}_{ab}\right) dx$ to $1 - \int_0^{2z-\frac{1}{2}} f\left(\frac{x}{z}, \frac{z-x}{z}; s\overline{v}_{ac}, s\overline{v}_{ab}\right) dx$. The analysis extends straightforwardly to all other first-round pivot events by similarly reversing inequalities and/or exchanging candidate labels.

Checking consistency of numerical and simulation-based estimates: To check the validity of the numerical approach and compare the computational burden of the two approaches, we computed pivotal probabilities for 100 scenarios using the two approaches while varying the number of simulation draws. If our numerical approach is correct, the simulation results should converge on our numerical solutions as the number of simulations (and the computational burden of the simulation approach) increases. Below we show that this is the case.

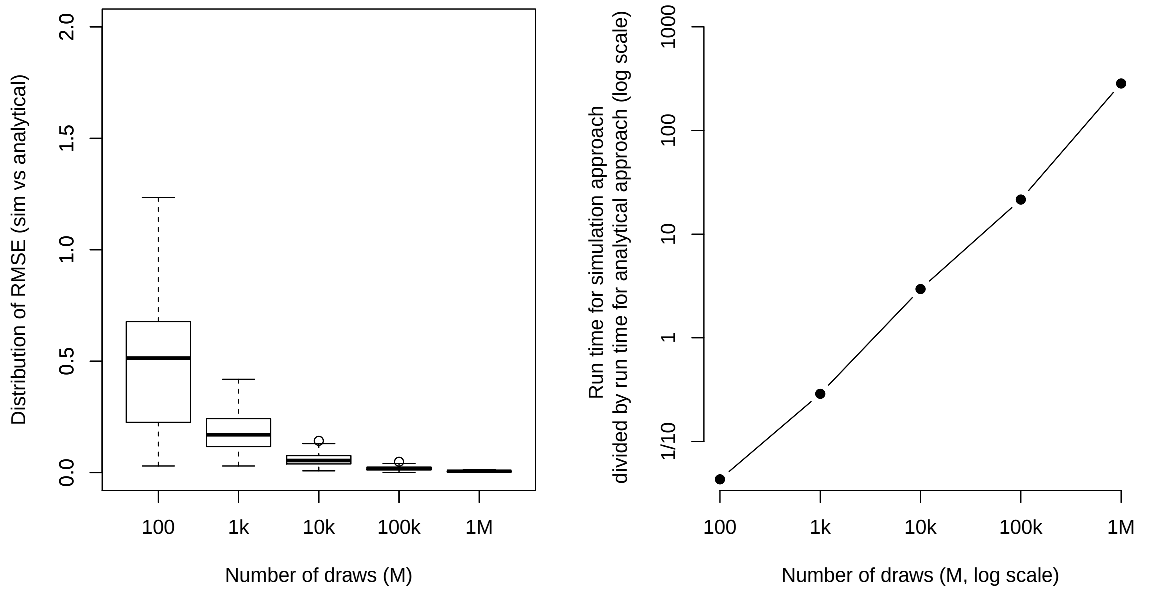
We begin by drawing J sets of Dirichlet parameter values at which we will calculate pivotal probabilities. Specifically, for scenario j we (1) draw a vector $\overline{\mathbf{v}}_j = (\overline{v}_{AB,j}, \overline{v}_{AC,j}, \overline{v}_{BA,j}, \overline{v}_{BC,j}, \overline{v}_{CA,j}, \overline{v}_{CB,j})$ from a Dirichlet distribution with parameters $(6, 4, 5, 5, 4, 6)$ and (2) draw s_j independently from a uniform distribution between 15 and 60. Together, $\overline{\mathbf{v}}_j$ and s_j define beliefs for scenario j. For each of these J scenarios there are 12 pivotal probabilities to compute. Let $\mathbf{T}$ denote the $J \times 12$ matrix of pivotal probabilities computed with our numerical approach, and let $\tilde{\mathbf{T}}_M$ denote the $J \times 12$ matrix of pivotal probabilities computed with our simulation method using M draws from the belief distribution. Our focus is on how the discrepancies between $\mathbf{T}$ and $\mathbf{T}_M$ vary with M. We summarize these discrepancies with two approaches.

First, for each M and for each of $J = 100$ we compute the root mean squared error (RMSE), or average discrepancy, between $\mathbf{T}$ and $\tilde{\mathbf{T}}_M$. That is, for a given M, we compute the RMSE for each row of $\mathbf{T}$ and $\tilde{\mathbf{T}}_M$. The left panel of Figure A.2 summarizes the distribution of these 100 RMSEs at each value of M. It shows that the distribution of RMSEs converges toward a point mass at zero as the number of draws from the belief distribution increases. As the simulation approach becomes more accurate, its computational burden also increases (as shown in the right panel): with M of 1 million, our machine takes over 250 times longer to compute the pivotal probabilities by simulation than by the analytical approach.[42]

Second, for each pivotal event we compute at each M the RMSE across the $J = 100$ scenarios between $\mathbf{T}$ and $\tilde{\mathbf{T}}_M$. That is, for a given M, we compute the RMSE for each column of $\mathbf{T}$ and $\tilde{\mathbf{T}}_M$. Figure A.3 summarizes how these RMSEs vary with M. It shows that the RMSE drops toward zero for all pivotal events as the number of number of draws from the belief distribution increases.

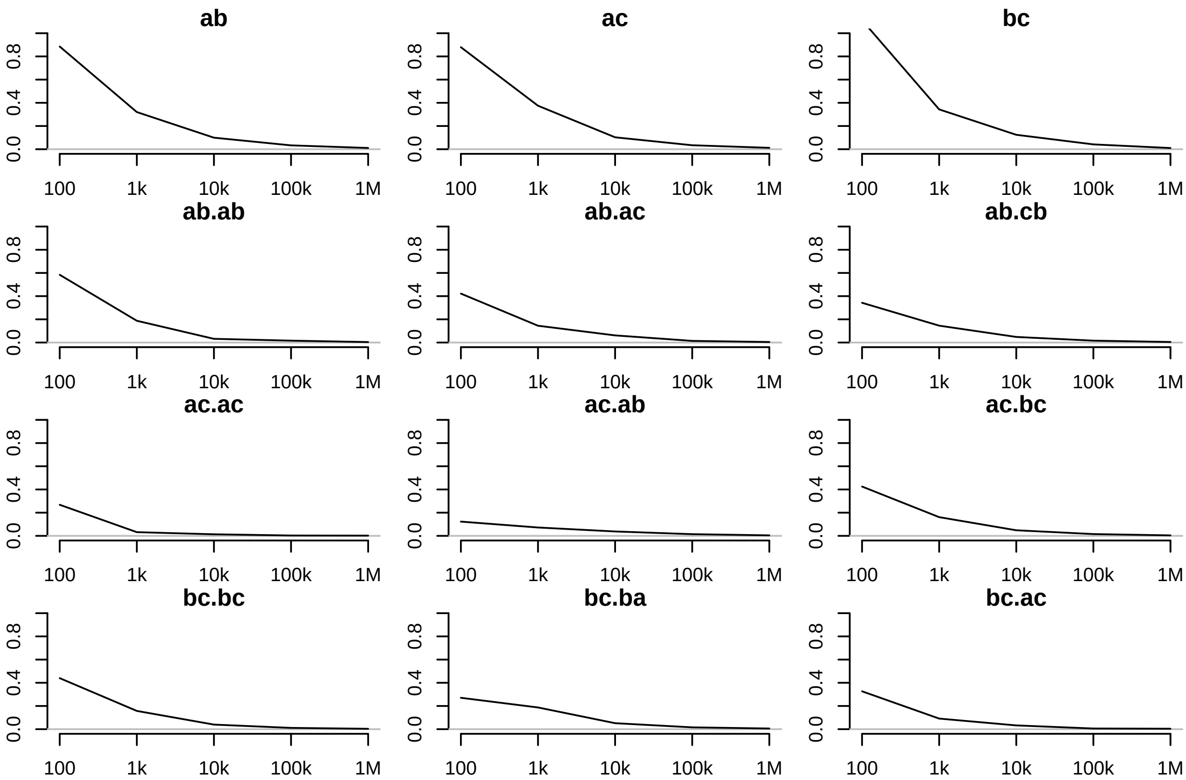
[42]Benchmarking performed on a 2017 MacBook Pro with 2.3 GHz processor and 16GB memory.

Figure A.2: Numerical/analytical approach agrees with simulations but is many times faster



Note: For each of 100 sets of belief parameters, we compute pivotal probabilities (1) analytically and (2) by simulation, with M draws from the belief distribution. We then calculate the RMSE across the 12 pivotal events between the analytical approach and the simulation approach for each of the 100 scenarios. The left figure shows, for each value of M (horizontal axis), that the distribution of the RMSEs across the 100 scenarios converges to a point mass at zero as the number of simulation draws increases. The right panel shows how the relative computational burden of the simulation approach increases as the number of simulation draws increases

Figure A.3: RMSE by pivotal event and number of draws in simulation



Note: For each of 100 sets of belief parameters, we compute pivotal probabilities (1) analytically and (2) by simulation, with M draws from the belief distribution. The figure shows, for each pivotal event, the average discrepancy (RMSE) between the two approaches as M increases.

A.3 Modeling beliefs about likely election outcomes

Finally, we provide greater detail about modeling beliefs about likely election outcomes. As stated in the main text, we rely on a Dirichlet distribution to describe voters' beliefs about the distribution of likely ballot shares. More formally, the distribution $f(\mathbf{v}_{-i})$ is defined as

$$f(\mathbf{v}_{-i}) = \mathrm{Dir}(\bar{\mathbf{v}} \times s) \tag{12}$$

where $\bar{\mathbf{v}}$ is the location parameter – describing the expected value of the distribution – and s is the scale parameter, scaling the uncertainty around that expectation. The choice of $\bar{\mathbf{v}}$ depends on our iterative polling algorithm. For the first iteration, we set the location parameter to correspond to the distribution of vote shares if everyone in the surveyed election had cast their ballot sincerely.

In each subsequent iteration m, the expected result at which the belief is centered is a weighted average of the previous expected result, $\bar{\mathbf{v}}_{\mathbf{m-1}}$, and voters' best response ($\bar{\mathbf{v}}^{\mathbf{BR}}$) to beliefs centered at $\bar{\mathbf{v}}_{m-1}$:

$$\mathbf{v}_{\mathbf{m}} = \lambda \mathbf{v}^{BR}(\bar{\mathbf{v}}_{m-1}, s) + (1-\lambda)\bar{\mathbf{v}}_{m-1}. \tag{13}$$

We run this algorithm for 250 iterations for each case. As the main text suggests, iterations at the beginning of the algorithm can be interpreted as assuming that most voters vote sincerely; while later iterations can be interpreted as assuming a highly strategic electorate.

B Selecting the optimal vote from expected utilities

In order to identify a voter's strategically optimal vote, we have to find the ballot choice that yields the highest expected utility. In technical terms, this is the row maximum of the matrix product of the utility matrix $\mathbf{U}$ and the pivotal probability matrix $\mathbf{P}$ (cf. Section 2 in the main paper). However, as some pivotal probabilities are of an extremely small magnitude (especially in IRV), we conventionally run into the 'floating point problem' when using computational approaches to identify the maximum (Goldberg, 1991). Essentially, finite memory means that computers can only store numbers with limited precision by approximating them to the closest defined floating point. As a consequence, tiny differences between numbers that lie inbetween two floating points are lost due to rounding. This problem affects the selection of row minima and maxima if the values are sufficiently small, and the difference between the two ballots under IRV with the same first preference depends on some very unlikely event.[43]

Ideally, we would increase the memory for each stored number, but this comes at a high computational cost. As a more feasible solution, we implement the following procedure to avoid selecting row maxima that are theoretically unjustified and only occur because of the floating point precision problem:

1. We add a very small value to the expected utility of voters' sincere votes, such that $\mathbf{EU} = \mathbf{UP}(\bar{\mathbf{v}}, s) + 10^{-10}\mathbf{S}$
2. Any remaining cases where two values in a row are seen as tied for maximum by the computer are resolved in favor of sincerity.

[43]For example, for a voter with sincere preference ABC, voting ABC or ACB amounts to the same except for the case where A is eliminated in the first round and the voter is pivotal between B and C in the second round.

C Convergence of the iterative polling algorithm

In this section of the appendix we provide additional evidence that (a) the iterative polling algorithm converges at all in IRV; (b) it converges onto a seemingly unique equilibrium of ballot shares. In plurality, we can infer the equilibrium behavior from the vote share paths in Figure 1 alone; as all voters with a sincere preference for C have responded by voting strategically for A or B, the result is a (quasi-)Duvergerian equilibrium and everyone's best response is to continue voting as they did in response to the previous poll.[44]

In IRV, we cannot make the same inference as there is no general (analytical) characterization of strategic voting equilibria. However, we provide evidence that the algorithm converges onto a unique ballot share, with the exception of oscillations as mentioned in Section 4.1 (for the least converged cases, see Appendix E to see that the patterns of change in best responses behave in regular patterns). We also provide evidence to suggest that this resulting equilibrium is robust to parameter choice (s, λ) as well as the 'starting point' of the equilibrium.

Notation. Let $\bar{\mathbf{v}}_{j,k}(s, \lambda)$ denote the (weighted) IRV ballot share vector (with six items) after the j^{th} iteration and in CSES case k. This vector is also the expected result upon which intial beliefs in the subsequent iteration $j+1$ are centered. For example, $\bar{\mathbf{v}}_{1,\text{AUS2013}}$ denotes the ballot shares at the end of the first iteration in the 2013 Australia case. This quantity, as defined by Equation 13, is a weighted average of voters' best response to initial beliefs in this iteration ($\mathbf{v}^{BR}_{j,k}(s, \lambda)$), and the initial expected result at the beginning of the iteration ($\bar{\mathbf{v}}_{j-1,k}(s, \lambda)$).

Next, let $d(\mathbf{m}, \mathbf{n}) = \sqrt{(\mathbf{m} - \mathbf{n})(\mathbf{m} - \mathbf{n})}$ denote the Euclidean distance between two arbitrary vectors of the same length. We then define, $D_{j,k}$ (the quantity in Figure 2 and Appendix C.1), as:

$$D_{j,k} = d(\bar{\mathbf{v}}^{BR}_{j,k}(s, \lambda),\ \bar{\mathbf{v}}_{(j-1,k)}(s, \lambda))$$

which is the Euclidean distance between the voters' best response to any given iteration in case k, and the ballot shares in the poll at the beginning of that iteration.[45]

Convergence onto a fixed point. In Appendix C.1, we report $D_{j,k}$ (the distance between voters' best response and the expected result at the beginning of the iteration), and show that the convergence behavior is robust to parameter choices.

Convergence onto an oscillating sequence. Appendix C.2 reports further context on the oscillating behavior under IRV and shows that when comparing the distance between an expected result and a lagged average of the algorithm output (smoothing any oscillation), the distance converges towards zero.

Convergence onto the same point across parameter values. In Appendix C.3, we provide evidence that the equilibria upon which the algorithm converges are robust to the parameter choice of λ.

Convergence onto the same point across starting points. Furthermore, Appendix C.4 suggests that the vote shares upon which the iterative polling algorithm converges in IRV may hold irrespective of the starting point. Together, these results characterize the behavior of the iterative polling algorithm under IRV and suggest that a general strategic voting equilibrium in IRV may exist.

[44] The same logic, merely with inverted party names, holds for the few cases where the eventual equilibrium pins A and C against each other.

[45] Alternatively, also denoted as the output of the algorithm in the iteration $j - 1$.

C.1 Euclidean distances between best response and ballot share vector

We check whether convergence of the polling algorithm also occurs under different parameter values for precision (s) and strategicness (λ). Analogous to Figure 2, we present, for each iteration j and every case k, $D_{j,k}$, the distance between the ballot shares of best responses given certain parameter values, and the ballot shares in the poll at the beginning of the algorithm (using the same parameter values). In contrast to Figure 2 in the main body, however, we use a logarithmic scale to plot the distance, in order to highlight changes of a small magnitude when the ballot shares do not move much anymore after multiple iterations.

In IRV (left panels), we see that for a handful of cases, the distances decrease continuously and evenly. The remainder sees their distance drop until the 50th or 60th iteration before stagnating at very small values ($e^{-7} \approx 0.0009$). This behavior occurs because of the oscillations, whereby a small number of voters changes their strategic vote in a regular pattern, thus preventing the algorithm from reaching a 'true' fixed point. In a setting with low strategic responsiveness (smaller values of λ), the distance decreases more slowly as convergence is slower; in most cases, 250 iterations are not sufficient to reach full convergence. In contrast, with high strategic responsiveness, the algorithm settles into a pattern where the poll-to-poll distances are greater, since more voters are part of the 'oscillation'. Although a few CSES cases are sensitive to the choice of s, the broader convergence pattern and magnitude of oscillation distances (conditional on λ) appear robust.

In Plurality, convergence occurs in a very regular and even fashion – there are no cases that get stuck in an oscillating pattern or stop converging towards zero. This corroborates theoretical knowledge about equilibria in Plurality. However, the speed of convergence and variance between cases is sensitive to values of λ and s.

C.1.1 Medium strategic responsiveness ($\lambda = 0.05$)

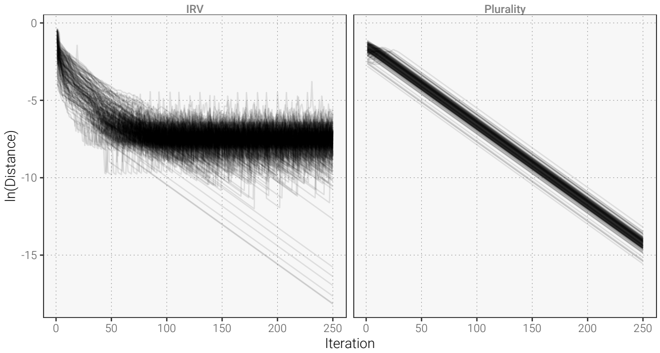


Figure A.4: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for high ($s = 85$) belief precision and medium ($\lambda = 0.05$) strategic responsiveness.

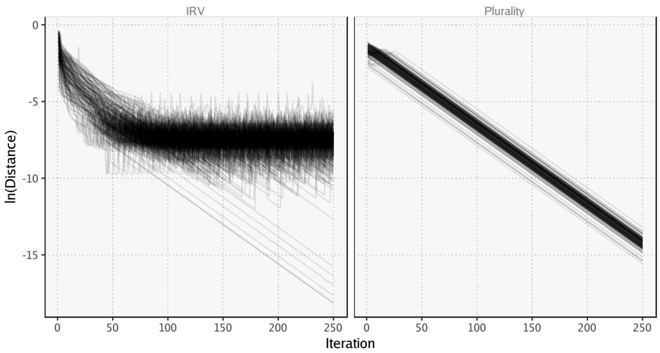


Figure A.5: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for low ($s = 10$) belief precision and medium ($\lambda = 0.05$) strategic responsiveness.

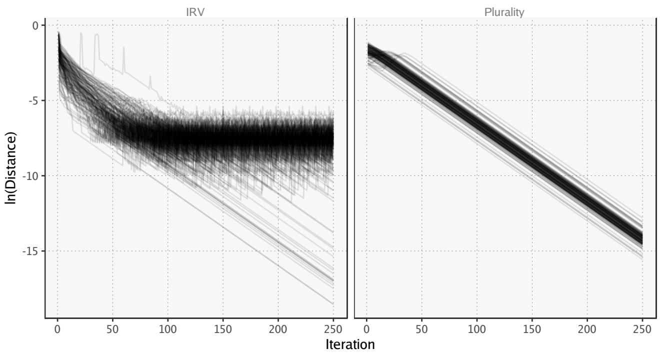


Figure A.6: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for medium ($s = 55$) belief precision and medium ($\lambda = 0.05$) strategic responsiveness.

C.1.2 Low strategic responsiveness ($\lambda = 0.01$)

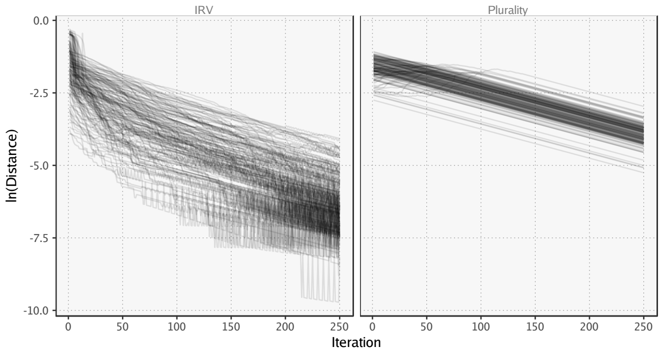


Figure A.7: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for high ($s = 85$) belief precision and low ($\lambda = 0.01$) strategic responsiveness.

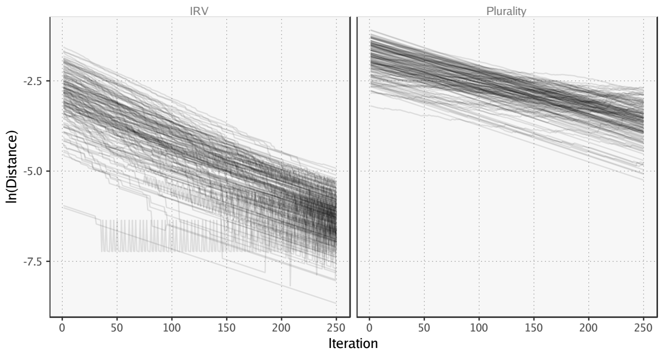


Figure A.8: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for low ($s = 10$) belief precision and low ($\lambda = 0.01$) strategic responsiveness.

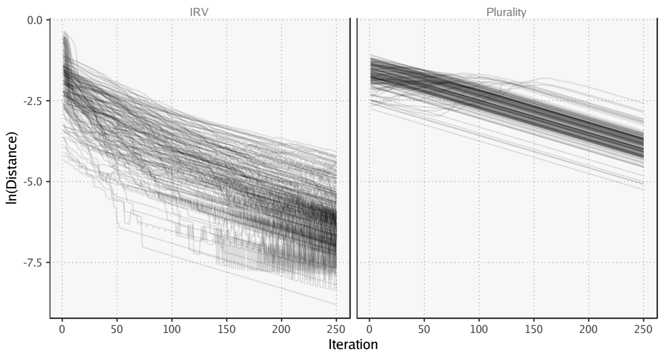


Figure A.9: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for medium ($s = 55$) belief precision and low ($\lambda = 0.01$) strategic responsiveness.

C.1.3 High strategic responsiveness ($\lambda = 0.10$)

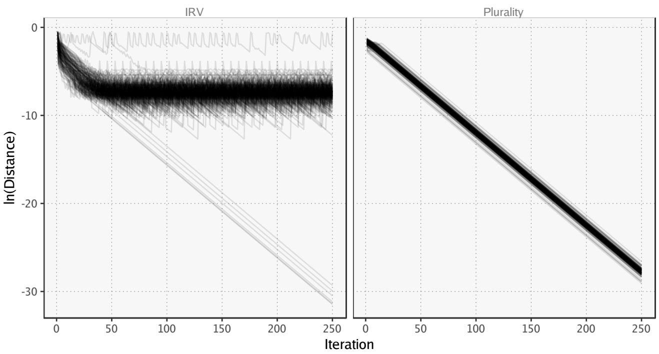


Figure A.10: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for high ($s = 85$) belief precision and high ($\lambda = 0.1$) strategic responsiveness.

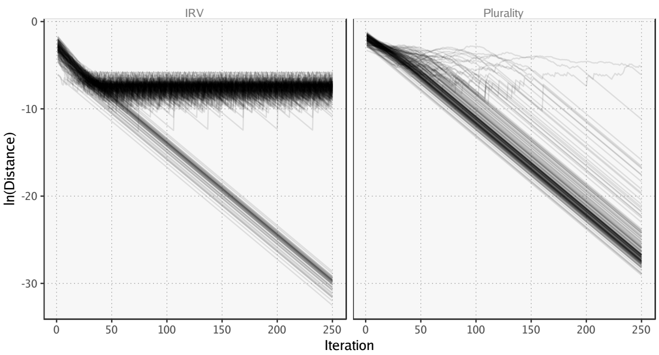


Figure A.11: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for low ($s = 10$) belief precision and high ($\lambda = 0.1$) strategic responsiveness.

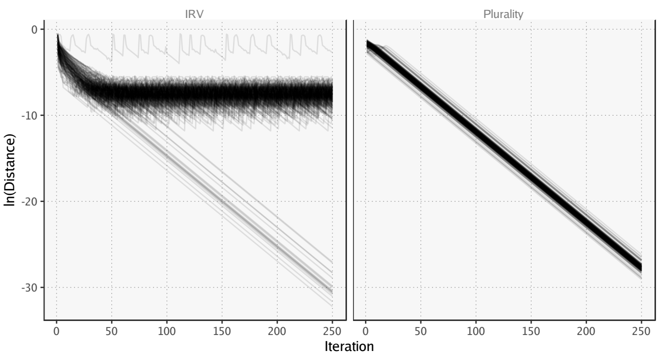


Figure A.12: Logged distance between poll result at the beginning of the iteration and voters' best response to it in IRV (left) and Plurality (right), and the vote shares in the previous iteration's poll. Results for medium ($s = 55$) belief precision and high ($\lambda = 0.1$) strategic responsiveness.

C.2 Euclidean distances between best response and lagged ballot share vector

To provide further context on the oscillating behavior under IRV, we report the Euclidean distance (Figure A.13) between the resulting best response ballot shares after the poll at time j of the algorithm, and the average of poll vote shares between times $j-20$ and $j-10$:

$$D^{lag}_{j,k} = d(\bar{\mathbf{v}}^{BR}_{j,k}(s,\lambda), \bar{\mathbf{v}}_{((t-10,\ t-20),\ k)}(s,\lambda))$$

Here, too, the quantity of interest decreases as voters become more strategic; the majority of cases settles in a band between 0 and 0.01.[46] This behavior indicates that although there are changes from one iteration to another due to a small number of voters changing their optimal strategic response, the overall vote share does not move in great distances across multiple iterations. Occasional spikes occur when that pattern is disrupted and the vote share moves a larger distance before settling into a new oscillation again. Altogether, the examination of vote share distances along the iteration paths suggests that in IRV, the algorithm settles on either a direct fixed point, or an oscillating pattern where only a small number of voters changes their strategic response in a regular manner.

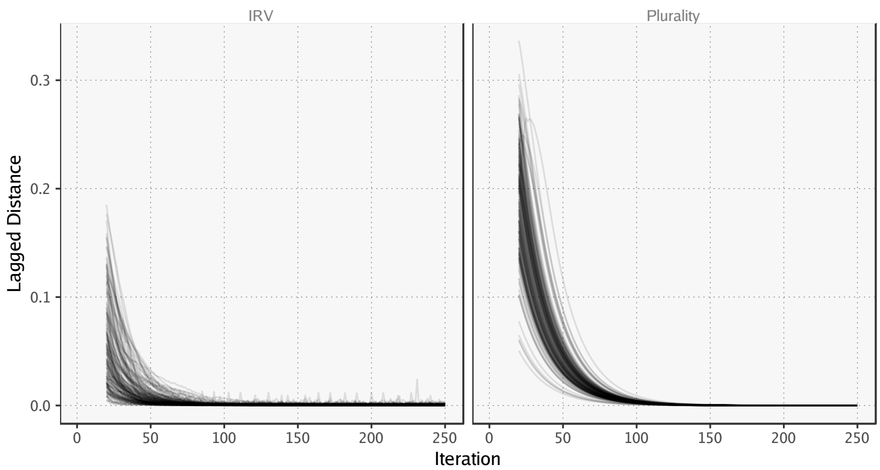


Figure A.13: Distance between the shares of voters' best responses after voters have been given a poll in IRV (with $\lambda = .05$ responding strategically), and the average of the respective vote shares 10 to 20 iterations ago.

[46]Note that for other parameter values [not shown], the range of this band will vary, but the general pattern holds.

C.3 Comparison of convergence paths relative to baseline case

We provide evidence that the equilibria upon which the algorithm converges are robust to the parameter choice of λ; we plot the distribution (across CSES cases) of distances between a jth poll with certain parameter values, and the resulting vote shares after the 250th poll in the baseline case ($s = 85$, $\lambda = 0.05$, Figure A.14), as well as after the 250th poll in the case with the same s, but holding $\lambda = 0.05$ (Figure A.15).

Formally, the quantities of interest are:

$$D^{base}_{j,k} = d(\bar{\mathbf{v}}_{j,k}(s, \lambda), \bar{\mathbf{v}}_{(250,\ k)}(s = 85, \lambda = 0.05))$$
$$D^{s-comp}_{j,k} = d(\bar{\mathbf{v}}_{j,k}(s, \lambda), \bar{\mathbf{v}}_{(250,\ k)}(s, \lambda = 0.05))$$

The results show that although the algorithm converges on different ballot shares conditional on the choice of s – the densities of distances do not converge onto zero when compared to the baseline of $s = 85, \lambda = 0.05$ (Figure A.14), the equilibrium is robust to the choice of λ: when comparing distances across different values of λ, but holding s fixed (Figure A.15), we see differences in how quickly the algorithm converges (which is what λ determines by definition), but, ultimately, the distances converge towards zero.

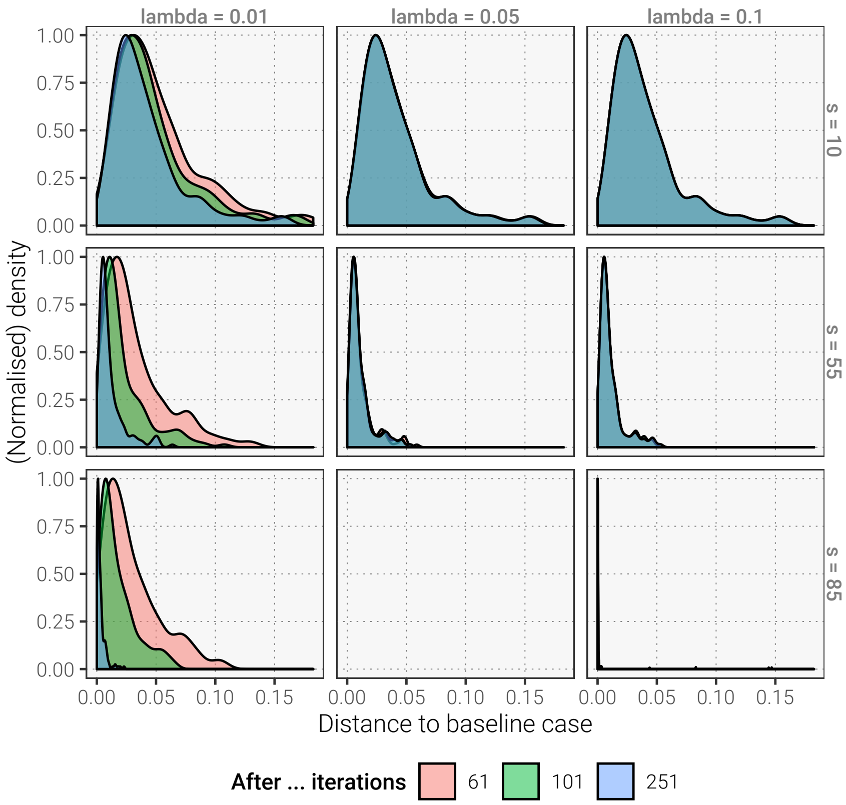


Figure A.14: Distribution of Euclidean distances across CSES cases between resulting vote shares in jth iteration under given parameter combination (information precision, s, and strategic responsiveness, λ) compared to 250th iteration in the baseline case ($s = 85$, $\lambda = 0.05$).

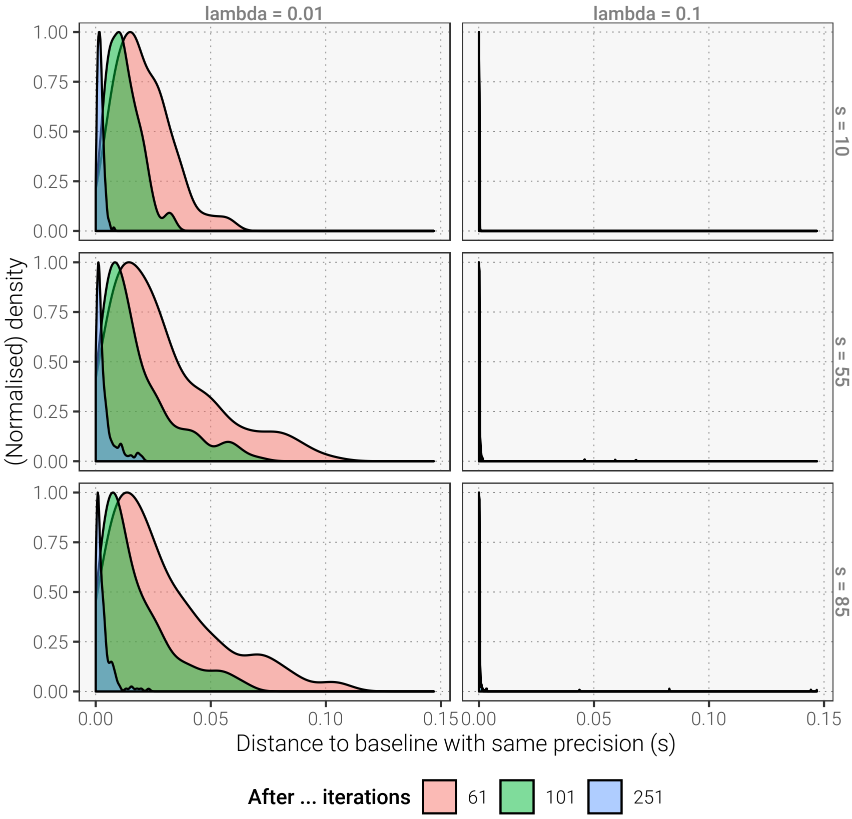


Figure A.15: Distribution of Euclidean distances across CSES cases between resulting vote shares in jth iteration under given parameter combination (information precision, s, and strategic responsiveness, λ) compared to 250th iteration in the case with same s but $\lambda = 0.05$.

C.4 Convergence under IRV with random starting points

In order to evaluate whether the CSES cases converge onto the same IRV strategic voting equilibrium irrespective of the initial belief about ballot shares, we draw 100 random ballot shares from a Dirichlet distribution with uniform density, and use these to initialize the polling algorithm. Let $q \in \{1, ..., 100\}$ denote the particular random draw. Formally, let $\tilde{\mathbf{v}}_{j,k}(s, \lambda, \tilde{\mathbf{v}}_0)$ denote the ballot share vector after j iterations for CSES case k, where the algorithm was initialized with the values s, λ, and a starting belief about ballot shares centered on $\mathbf{v}_0$. Then, the "random starting point distance to baseline case" refers to the distance between the ballot share vector after the jth iteration for case k and a random starting belief centered on $\mathbf{v}_q$, and the ballot share after the 250th iteration where the algorithm was initialized with baseline parameter values ($s = 85, \lambda = 0.05$), and the sincere ballot share profile for that case. Formally,

$$\tilde{d}_{j,k,q} = d(\tilde{\mathbf{v}}_{j,k}(s, \lambda, \bar{\mathbf{v}}_q), \mathbf{v}_{250,k}(s = 85, \lambda = 0.05, \mathbf{v}_{true}))$$

Figure A.16 summarises the distribution of distances between ballot shares starting at random points (with $s = 85, \lambda = 0.05$, i.e., baseline parameter values), and the 'converged' ballot share after 250 iterations starting at each case's sincere profile. Each point indicates the median, 90th or 99th quantile of the distribution of distances (y-axis) between the algorithm from random starting points and the converged ballot shares (after 250 iterations) coming from the sincere voting profile, for each case and after each iteration (x-axis). Formally, each point represents a summary statistic of all $\tilde{d}_{j,k,q}$ for each case k, and after each iteration j across all 100 random draws.

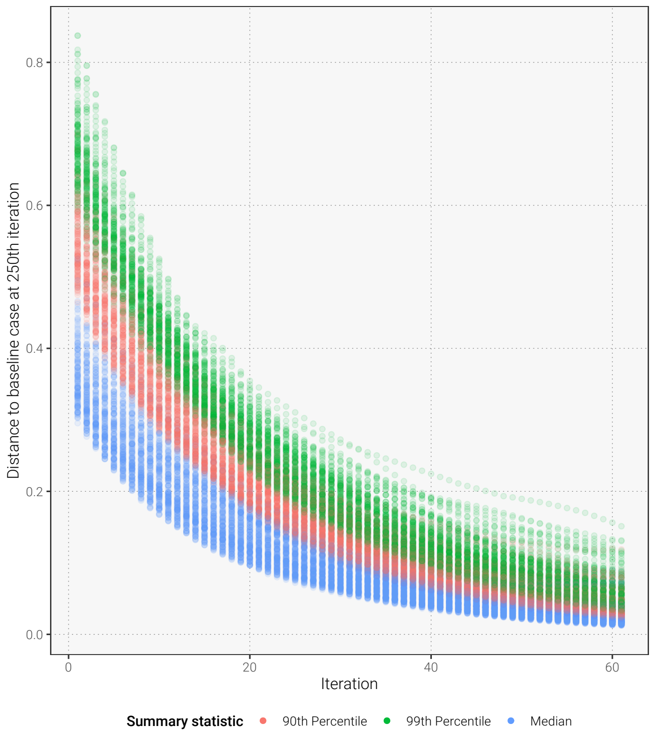


Figure A.16: Summary of distances between case-specific distributions of distances between ballot shares after iterations from random starting points, and the converged ballot shares in the baseline case

D Robustness of expected benefit results to number of iterations

In this section, we present results from Figure 4 extended to 250 (rather than 60) iterations. The results do not change substantially beyond 60 iterations.

D.1 Medium strategic responsiveness ($\lambda = 0.05$)

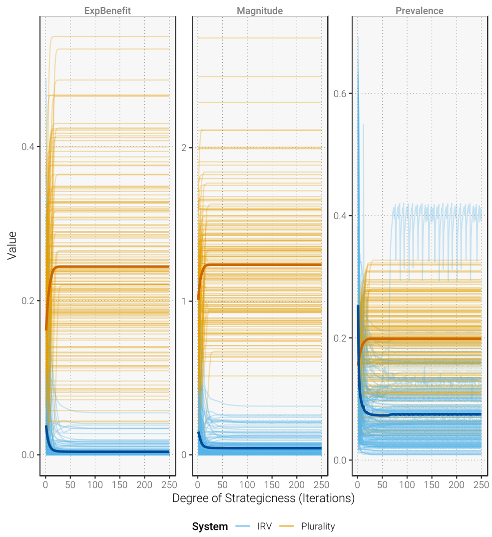


Figure A.17: Expected benefit, magnitude, and prevalence of strategic voting with high ($s = 85$) belief precision, and medium strategic responsiveness ($\lambda = 0.05$).

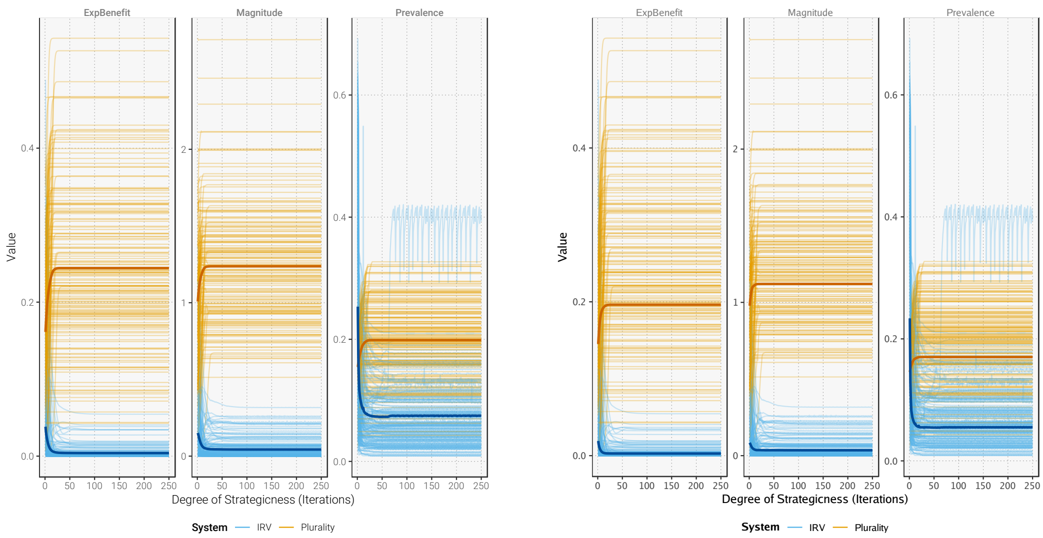


Figure A.18: Expected benefit, magnitude, and prevalence of strategic voting with low ($s = 10$, left) and medium ($s = 55$, right) belief precision, and medium strategic responsiveness ($\lambda = 0.05$).

E Case-specific strategic voting behavior

Finally, Appendix E offers a case-by-case examination of the prevalence of strategic voting for all cases where the distance between the best response and the outcome of the previous iteration in the baseline case never falls below the value of 0.001.[47]

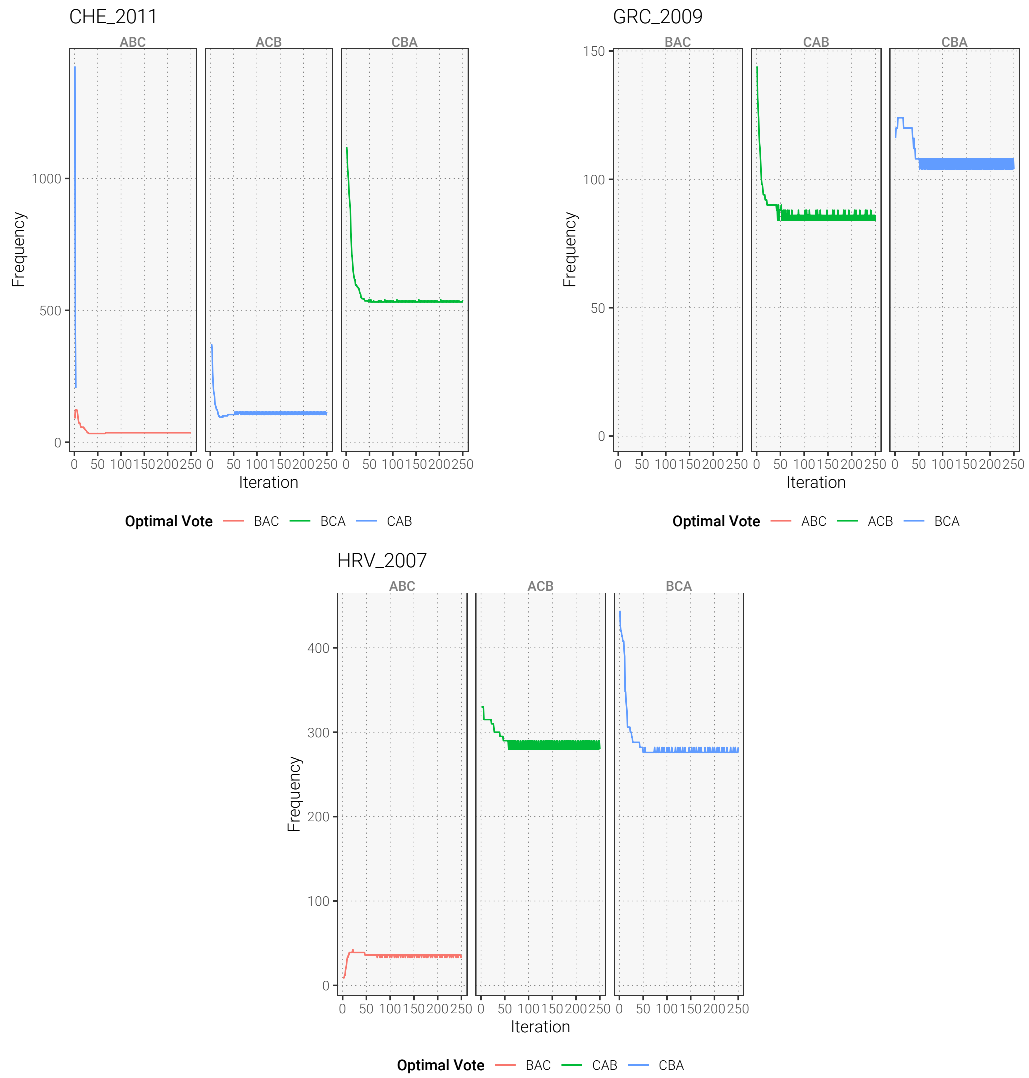


Figure A.19: Frequency of optimal strategic votes by case and voter type (sincere preferences) under IRV, as the polling iteration algorithm proceeds. Selected cases shown whose iteration-to-iteration Euclidean distance between vote shares did not converge below a threshold of 0.001 in the baseline case of $s = 85$ and $\lambda = 0.05$. Cases are Czech Republic 2011 (top left), Greece 2009 (top right) and Croatia 2007 (bottom).

[47] Appendix E only reports the three qualifying cases obtained when $s = 85$ and $\lambda = 0.05$.